International Academic Institute for Science and Technology

International Academic Journal of Science and Engineering
Vol.11, No.1, 2024, pp.15-18.

ISSN 2454-3896

International Academic Journal of Science and Engineering

www.iaiest.com

Fuzzy Nbhd System in Fuzzy Top-R-Module

Basim Mohammed Melgat [a]

[a] *Ph.D., Department of Civil Engineering, AL-Qasim Green University College of Engineering, Iraq.*
E-mail: Dr.basimmohammed@werc.uoqasim.edu.iq

Abstract

We will give a crucial and suitable condition for a family of fuzzy sets in a module M to be the family of neighborhoods of a member m_α in fuzzy top. module.

Keyword: Fuzzy Top-R-module, Fuzzy Nbhd System., Fuzzy Open Set, Fundamental System of Fuzzy Nbhds.

DOI:10.71086/IAJSE/V11I1/IAJSE1103
Received:30 November 2023/Accepted: 28 December 2023/Published: 24 January 2024

Introduction

Lutfy. Z. (1965) [10], basid the fuzziness. Chang (1968,) [1] defined fuzzy top. Deb R., A. [5,6] defined fuzzy of top. ring, continuous function and left top. ring. Basim M. M. and M. AL-Khafaji [2, 3 and 4] (2019) gave some results of fuzzy top. ring. In 2020 [7] Mohammed M. A., studied fuzzy top. module and fuzzy topological submodule.

We give a new study of fuzzy nbhds of fuzzy point $m_\alpha \in M$. we explain for this collection there is an only one fuzzy top. module. let M has one or more than one element.

Definition 1.1 [9]

The map $A: M \to J$ ($J = [0,1]$) is a fuzzy set A in M, this means a member of J^M . If $A \in J^M$. $m \in M$, then $A(m)$ or m_α is a membership of m in A .

Definition 1.2 [1]

A family $\tau_F \in J^M$ of fuzzy set is a fuzzy top. for M if the next conditions are hold:

1) $\emptyset, M \in \tau_F$
2) $\forall E, B \in \tau_F \to E \wedge B \in \tau_F$
3) $\forall (E_j)_{j\in J} \in \mu \to \vee_{j\in J} E_j \in \tau_F$

(M, τ_F) is a fuzzy top. space and E is fuzzy open if $E \in \tau_F$. and E^c is a fuzzy closed.

Definition 1.3 [7]

Assume R is a ring and M is a left R-module. A fuzzy set E in M is a fuzzy left R-module if $\forall\, m, n \in M \; and \; r \in R$:

(1) $E(m+n) \geq min\{E(m), E(n)\}$.
(2) $E(m) = E(m^{-1})$.
(3) $E(rm) \geq E(m)$.
(4) $E(0) = 1$.

Definition 1.4 [7]

Assume R is a fuzzy top. ring then M is called left fuzzy top. module if:

(1) E left fuzzy module on R.
(2) E is a fuzzy top. group on M and the function $R \times M \longrightarrow M$, $f(r,m) = r.m$, ($r \in R \; and \; m \in M$) is a fuzzy conts.

Proposition 1.5 [3]

Assume $\{U_0\}$ is the system of every fuzzynbhds of 0 in a fuzzy top. ring (R, τ_R) . Then for any fuzzy subset E of R,

$$cl(E) = \cap(E + U)$$

Theorem 1.6 [4]

Let (R, τ_R) be an fuzzy top. ring. and let $\{U_0\}$ The family of all fuzzy open nbhds of 0 . Then $\{U_0\}$ has the following properties:

1) $m_\alpha \in \cap\,(U + V)$
2) If $U_1, U_2 \in \{U_0\}$ then $U_1 \cap U_2 \in \{U_0\}$.
3) If $U \in \{U_0\}$, then $-U \in \{U_0\}$.
4) If $U \in \{U_0\}$, then $U + U \in \{U_0\}$.
5) if $m \in M$, then $U_m = \{m + V : V \in U_0\}$ is a fuzzy base of nbhds of m_α.

Definition 1.7[8]

A fuzzy set E in (M, τ_M) is a fuzzy nbhd of $m \in M$ iff $\exists U \in \tau_M$ s.t $\alpha \leq U(r) \leq E(r)$. U is a fuzzy open set iff it's a fuzzy nbhd of each fuzzy point of its.

Fuzzy Neighborhood Systems

Theorem 2.1

Assume (R, τ_R), (M, τ_M) is a fuzzy (top. ring, top-R-module) consequently, $m \in M$, $r \in R$ s.t $m_\alpha \in \{n : M(n) = \max\{M(h)\}, \forall h \in M\}$, then

1) $f_{r_\beta}: (M, \tau_M) \to (M, \tau_M), f_{r_\beta}(m_\alpha) = r_\beta . m_\alpha$ is a fuzzy cont. mapping
2) $f_{m_\beta}: (R, \tau_R) \to (M, \tau_M), \quad f_{m_\beta}(m_\alpha) = r_\beta . m_\alpha$is fuzzy cont. mapping

Proof

Result from definition of fuzzy top-R-module 1.4

Corollary 2.2

Assume (R, τ_R) is a fuzzy top. commutative ring with a unitary element and (M, τ_M) is fuzzy top-R-module. Let $m \in M$ an invertible element, $m_\alpha \in \{n: M(n) = \max\{M(h)\}, \forall h \in M\}$ and $r \in R$, so

1) $f_{r_\beta}: (M, \tau_M) \to (M, \tau_M), f_{r_\beta}(m_\alpha) = r_\beta . m_\alpha$ is a fuzzy homeo. mapping
2) $f_{m_\beta}: (R, \tau_R) \to (M, \tau_M), \quad f_{m_\alpha}(r_\beta) = r_\beta . m_\alpha$is fuzzy homeo. mapping

Corollary 2.3

Suppose (M, τ_M) be a fuzzy top-R-module space and $E \leq \tau_M$. Suppose $m_\alpha \in \{n_\alpha : M(n) = \max\{M(h)\}, \forall h \in M\}$. If E is fz-open (fz-closed) set then $m.E$ and $E.m$ are fuzzy open (fuzzy closed set).

Theorem 2.4

Let (M, τ_M) be a fuzzy top-R-module. space and $\{U_0\}$ a fuzzy fundamental system of 0. Then for any fuzzy subset E of M, $m_\alpha \in cl(E)$ iff $m_\alpha \in \bigwedge(E + U)$, where

$$m_\alpha \in \{n_\alpha: M(n) = \max\{M(h)\}, \qquad \forall h \in M\}$$

Proof

Since (M, τ_M) a fuzzy top-R-module. space, then $(M, \tau_M)$is a fuzzy top. abelian group then by Proposition 1.5 we get $m_\alpha \in cl(E)$ iff $m_\alpha \in \bigwedge(E + U)$

Theorem 2.5

Suppose (M, τ_M) be a fuzzy top-R-module and let $\{U_0\}$ be a fundamental system of fuzzynbhds of 0 such that $U(0) = \max\{M(n)\}, \forall n \in M\}$ of fz-top R-module, then the following statement are true

1) $m_\alpha \in \bigwedge(U + V)$
2) for any $U \in U_0$ and $V \in U_0$ there exist $W \in U_0$ s.t $W \leq V \cap U$.
3) for any $U \in U_0$ there exist $V \in U_0$ s.t $V + V \leq U$.
4) for any $U \in U_0$ there exist $V \in U_0$ s.t $-V \leq U$.
5) if $m_\alpha \in M$, then $U_m = \{m + V: V \in U_0\}$ is a fuzzy fundamental system m_α.
6) $\forall\, U \in U_0$ t$\exists\, V \in U_0$and a fuzzynbhd W of 0 in R s.t $W.V \leq U$.
7) for any $U \in U_0$ and $r \in R$ there exist $V \in U_0$ s.t $r.V \leq U$.
8) for any $U \in U_0$ and $m \in M$ there exist a fuzzynbhd V of 0 in R s.t $V.m \leq U$.

Proof

Result of 1,2,3,4 and 5 By theorem 1.6

6) Let U be an fuzzynbhd of 0 and $U(0) > 0$, then $\exists V \in \tau_M$, $V(0) > 0$, $V \leq U$ and $V(0) = U(0) = \max\{R(h)\}$, $\forall h \in R\}$. Since (M, τ_M) is a fuzzy top-R-module space, then there exists a fuzzynbhd W of 0 in R and fz-open set $V_1 \in \tau_M$ s.t $f(W \times V_1) \leq W.V_1 \leq W.V \leq W.U$ implies $W.U \in \{U_0\}$

(7) Let U be an fuzzynbhd of 0 and $U(0) > 0$, then $\exists V \in \tau_M$, $V(0) > 0$ s.t $V \leq U$ and $V(0) = U(0) = \max\{R(h)\}$, $\forall h \in R\}$. let $f_{r_\beta} : M \to M$; $f_{r_\beta}(m_\alpha) = r_\beta . m_\alpha$, Since (M, τ_M) is a fuzzy top-R-module space, then the mapping f_{r_β} is fuzzy homeo. [corollary 2.2.a] and therefore $r.V$ is fuzzy open set of 0 [corollary 2.3]. Hence, there exists fuzzy open set V_1 with $V_1 \leq {f_{r_\beta}}^{-1}(rV)$. Thus $rV_1 \leq rV \leq rU$

(8) Let U be an fuzzynbhd of 0 and $U(0) > 0$, then $\exists V \in \tau_M$, $V(0) > 0$ s.t $V \leq U$ and $V(0) = U(0) = \max\{R(h)\}$, $\forall h \in R\}$. let $f_{m_\alpha} : R \to M$; $f_{m_\alpha}(r_\beta) = r_\beta . m_\alpha$, Since (M, τ_M) is a fuzzy top-R-module space, then the mapping f_{m_α} is fuzzy homeo. [corollary 2.2.b] and therefore $V.m$ is fuzzy open set of 0[corollary 2.3]. Hence, there exists fuzzy open set V_1 with $V_1 \leq {f_{m_\alpha}}^{-1}(r.V)$. Thus $V_1.m \leq V.m \leq U.m$

Theorem 2.6

Assume M is a fuzzy R-module and $m_\alpha \in M$ be an invertible fuzzy point s.t $m_\alpha \in \{n_\alpha : E(n) = \max\{E(h)\}, \forall h \in R\}$. Let $\{U_m\}$ be a fuzzy fundamental open nbhds of m_α hold thm. 2.5. Then there is only one fuzzy top. τ_M s.t (M, τ_M) is fuzzy top-R-module.

Proof

Clearly that (M, τ_M) is a unique fuzzy top-R-module. We claim the following map $f: R \times M \longrightarrow M$ defined by $f(r_\beta, m_\alpha) = r_\beta . m_\alpha$ is a fuzzy continuous

Now assume$U_{r_\beta . m_\alpha .}$be a fuzzynbhd.. $\exists U \in \{U_m\}$ s.t $f^{-1}(U)(r_\beta, m_\alpha) = U\left(f(r_\beta . m_\alpha)\right) = U(r_\beta . m_\alpha) > 0$

since $U \in \{U_m\}$, by Thm. 2.5 $\exists V \in U_m$and a fuzzynbhd W of r in R s.t $W.V \leq U$

Thus $f: (R, \tau_R) \times (M, \tau_M) \to (M, \tau_M)$,$f(r_\beta . m_\alpha) = r_\beta . m_\alpha$ is fuzzy continuous

References

Chang, C.L. (1968). Fuzzy topological spaces. *Journal of mathematical Analysis and Applications*, *24*(1), 182-190.

Basim Mohammed Melgat & Munir AL-Khafaji (2019). Fuzzy Topological Rings Induced by Fuzzy Pseudo Normed Ring. *Jour of Adv Research in Dynamical & Control Systems, 11*(7) Special Issue.

Melgat, B.M., & AL-Khafaji, M.A.K. (2019). Some Result of Fuzzy Separation Axiom in Fuzzy Topological ring Space. *MJPS*, *6*(2), 13-19.

Melgat, B.M., & AL-Khafaji, M.A.K. (2019). Fuzzy Neighborhood Systems in Fuzzy Topological Ring. *Journal of Al-Qadisiyah for computer science and mathematics*, *11*(4), 87-93.

Deb Ray, A & Chettri, P (2009). On Fuzzy Topological Module Valued Fuzzy Continuous Functions. *Applied Mathematical Sciences, 3*(24), 1177-1188.

Deb Ray, A (2011). On (left) fuzzy topological module. *Int. Math.*, *6*(25-28), 1303-1312.

Al-Shamiri, M.M.A. (2020). Some Results on Fuzzy Topological Modules and Fuzzy Topological Submodules. *Asian Journal of Fuzzy and Applied Mathematics*, *8*(01).

Warren, R.H. (1978). Neighborhoods, bases and continuity in fuzzy topological spaces. *The Rocky Mountain Journal of Mathematics*, 459-470.

Zadeh, L.A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353.