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Innovative Acceleration Methods to Numerically Optimize the Values of Integrals with Simpson Rule 3/8

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Abstract

The main goal of this research is to present innovative acceleration methods that are very useful in improving the numerical results of definite integrals of continuous integrals, in which the main error is of the fourth order - in terms of accuracy, the number of subintervals used, and the speed of obtaining results, specifically to accelerate the results obtained by the Simpson 3/8 method.

Keywords: Acceleration Methods, Inverse Tangent Triangle Law of Acceleration, One-sided Integrals, for Continuous Integrals, Simpson's Three-eighth Rule.

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Introduction to Research

There are numerical methods for calculating single integrals defined in their integral intervals.

Trapezoidal Rule

Midpoint Rule

Simpson's Rule

Simpson's Rule 8/3, Which are called Newton-Cotes formulas.

In this research, we will discuss the Simpson 3/8 method for finding approximate values for one-sided integrals. Continuous Integrands using Al-Tamimi's Triangular Acceleration Methods. We will compare these methods with each other to choose the best one in terms of accuracy and speed of approaching the values of those methods to the true (analytical) values of those integrals. Let us assume the integral J is defined as follows:

$$J = \int_{x_0}^{x_3} f(x)dx \qquad (1.1)$$

Where f(x) is a continuous integrator located above the x axis in the interval [x_o, x_3] and what is required is to calculate the value of J in an approximate manner. In general, the Newton-Coates integral formulas (1) can be written in the following form:

$$\mathrm{J} = \int_{x_0}^{x_3} f(x)dx = S^{\$}(h) + E_{S^{\$}(h)} + R_{\$} \qquad (1.2) \qquad [1]$$

Where $S^{\$}$(h) represents the Lagrangian - Approximation for the value of the integral J, $E_{S^{\$}(h)}$ is the series of correction terms that can be added to the values of $S^{\$}$ (h) and $R_{\$}$ is the residual (Remainder) The value of S* (h) for Simpson's rule 3/8, which we denote by the symbol S^$ (h), is:

$$S^{\$}(h) = \frac{3h}{8}[f(a) + 3f(a+h) + 3f(a+2h) + 3f(a+3h) + 3f(a+4h) \ldots + 2f(n-3h) + 3f(n-2h) + 3f(n-h) + f(n)] \qquad \ldots (1.3)$$

The general formula for $E_{S^{\$}}(h)$ is as follows.

$$E_{S^{\$}(h)} = \frac{3(\mathrm{X_3} - \mathrm{X_0})}{80}\, \mathrm{h^4 f^{iv}}(\bar{\mathrm{x}}) \qquad [2]$$

Thus, when [3] the integrator is a continuous function and its derivatives are present at every point of the integration period [x_0, x_3], we can write the error formula.

$$\mathrm{E} = \mathrm{I} - \mathrm{S}^{\$}(\mathrm{h}) \cong \text{Ɏ}_1\mathrm{h}^4 + \text{Ɏ}_2\mathrm{h}^6 + \text{Ɏ}_3\mathrm{h}^8 + \ldots$$

Whereas, Ɏ_2, Ɏ_3,... Ɏ_1 are constants whose values do not depend on h, but rather depend on the values of the derivatives of the function at the ends of a period.

The integral and J are the real (exact) value of the integral.

Acceleration of Inverse Trigonometric Functions According to Simpson's Rule

$$\mathrm{E} = \text{Ɏ}_1\mathrm{h}^4 + \text{Ɏ}_2\mathrm{h}^6 + \text{Ɏ}_3\mathrm{h}^8 + \ldots \quad (2.1)$$

$$\mathrm{E} = \mathrm{h}^3(\text{Ɏ}_1\mathrm{h} + \text{Ɏ}_2\mathrm{h}^3 + \text{Ɏ}_3\mathrm{h}^5 + \ldots) \cong \mathrm{h}^3 \sin^{-1}(\mathrm{h})$$

$$\text{since } (\sin^{-1}(\mathrm{h}) = \mathrm{h} + \mathrm{h}^3 + \mathrm{h}^5 + \cdots) \qquad \ldots \quad (2.2) \quad [4]$$

Assuming that $S^{\$}(h)$ is the approximate value of the integral, Simpson's rule 8 3/then,

$$\mathrm{I} - \mathrm{S}^{\$}(\mathrm{h}) \cong \mathrm{h}^3 \sin^{-1}(\mathrm{h}) \qquad \ldots (2.3)$$

If we assume that we calculated two values for J numerically using Simpson's rule 3/8, they are $S^{\$}(h_1)$ when,

$$\mathrm{h} = \mathrm{h}_1 \; \text{ƛ} S^{\$}(h_2)$$

When h=h_2 then,

$$\mathrm{I} - \mathrm{S}^{\$}(\mathrm{h}_1) \cong \mathrm{h}^3 \sin^{-1}(\mathrm{h}_1) \qquad (2.4)$$

$$\mathrm{I} - \mathrm{S}^{\$}(\mathrm{h}_2) \cong \mathrm{h}^3 \sin^{-1}(\mathrm{h}_2) \qquad (2.5)$$

From formulas (3.4) and (3.5) we get,

$$\text{Æ}^{\mathrm{S}^{\$}}{}_{\sin^{-1}} \frac{(h^3{}_1 \sin^{-1} h_1)\, CS(h_2) - (h^3{}_2 \sin^{-1} h_2)\, CS(h_1)}{h^3{}_1 \sin^{-1} h_1 - h^3{}_2 \sin^{-1} h_2} \qquad \ldots (2.6)$$

We call formula (2.6) the inverse sine acceleration law $Æ^{S\$}{}_{\sin^{-1}}$.

In the same manner as above, we obtain the second acceleration triangle law:

$$Æ^{S\$}{}_{\cos^{-1}} \frac{(h^4{}_1\ cos^{-1} h_1)\ CS(h_2) - (h^4{}_2 cos^{-1} h_2)\ CS(h_1)}{h^4{}_1\ cos^{-1} h_1 - h^4{}_2 cos^{-1} h_2} \quad \dots (2.7)$$

We call formula (2.7) the inverse cosine acceleration law $Æ^{S\$}{}_{\cos^{-1}}$.

Likewise, the law can be written as the third triangular acceleration, and we will call it the inverse tangent triangle acceleration law and symbolize it $Æ^{S\$}{}_{\tan^{-1}}$ so the error E above can be written as follows:

$$E = \Upsilon_1 h^4 + \Upsilon_2 h^6 + \Upsilon_3 h^8 + \dots \quad \dots (2.8)$$

$$E = h^3(\Upsilon_1 h + \Upsilon_2 h^3 + \Upsilon_3 h^5 + \dots) \cong h^3 \tan^{-1}(h)$$

$$\text{since } (\sin^{-1}(h) = h + \frac{h^3}{3} + \frac{h^5}{5} + \cdots) \quad [4]$$

$$Æ^{S\$}{}_{\tan^{-1}} \frac{(h^3{}_1\ tan^{-1} h_1)\ CS(h_2) - (h^3{}_2\ tan^{-1} h_2)\ CS(h_1)}{h^3{}_1\ tan^{-1} h_1 - h^3{}_2\ tan^{-1} h_2} \quad \dots (2.9)$$

Examples

Based on the above, we will discuss some integrals whose integrals are continuous in the integration period, and we will use trigonometric acceleration methods to improve the results for the integrals below.

1. $I = \int_3^4 \frac{1}{x} dx - 1$ and its analytical value is 0.28768207245178, which is rounded to fourteen decimal places.
2. $l = \int_9^{10} \frac{1}{\sqrt{1+x}} dx - 2$ 〗 and its analytical value is 0.30869426037404, which is rounded to fourteen decimal places.
3. $I = \int_7^8 \frac{1}{X^{\frac{1}{2}} + X^{\frac{1}{3}}} dx - 3$ and its analytical value is 19106335261395 0, which is rounded to fourteen decimal places.

Accordingly, the tables for the six expedited methods are compared once with Simpson's three-eighth rule and once with each other, and the preference for the expedited method is calculated on the basis of the values of n, n=3, 6, 9..., and the results that we adopted in the MATLAB program are set as Eps=12-10.

Results

1. The integrator $I = \int_3^4 \frac{1}{x} dx$ is continuous in the integration period ⌈3,4⌉ and the formula for the correction limits for Simpson's rule is 3/8 as in formula No. (1.3).

Table 1: Calculating the Integral $I = 0.28768207245178$ Numerically Using Simpson's Three-eighth Rule with Acceleration of Trigonometric Functions

N	Values of simpson's 3/8 rule	$Æ^{S\$}{}_{\cos^{-1}}$	$Æ^{S\$}{}_{\sin^{-1}}$	$Æ^{S\$}{}_{\tan^{-1}}$
3	0.28768939393939			
6	0.28768255247803	0.28768209547621	0.28768209638267	0.28768209637649
9	0.2876821681773	0.28768207350728	0.28768207358028	0.28768207358001
12	0.28768210284254	0.28768207258821	0.2876820726019	0.28768207260186
15	0.28768208491946	0.28768207248112	0.28768207248503	0.28768207248502
18	0.28768207846952	0.28768207246032	0.28768207246176	0.28768207246175
21	0.28768207570169	0.28768207245482	0.28768207245544	0.28768207245544
24	0.28768207435746	0.28768207245303	0.28768207245333	0.28768207245333
27	0.28768207364176	0.28768207245235	0.28768207245251	0.28768207245251

This example was solved in the thesis (Al-Tamimi's Innovative Acceleration Methods of the Second Class to Improve the Results of Integral Values Numerically) gave accuracy using the Simpson method without accelerating to eight decimal places, while using accelerations we obtained integer values up to twelve decimal places, while our method obtained eight decimal places using the Simpson method. Three eighths without acceleration, while using acceleration we obtained integer values up to [6] [5]. n=27 when the decimal is of the order of ten, twelve.

2. The integral l=∫_(9)^10 $l = \int_9^{10} \frac{1}{\sqrt{+x1}}$dxis continuous in the integration period [9,10] and the formula for the correction limits is for Simpson's rule 3/8 as in formula No. (1.3).

Table 2: Calculating the Integral $l = \int_9^{10} \frac{1}{\sqrt{1+x}}$ dx $= 0.30869426037404$ Numerically Using Simpson's Three-eighth Rule with Acceleration of Trigonometric Functions

N	Values of simpson's 3/8 rule	$Æ^{S\$}{}_{\cos^{-1}}$	$Æ^{S\$}{}_{\sin^{-1}}$	$Æ^{S\$}{}_{\tan^{-1}}$
3	0.30869428617446			
6	0.30869426199376	0.30869426037851	0.30869426038172	0.3086942603817
9	0.30869426069425	0.30869426037413	0.30869426037437	0.30869426037437
12	0.30869426047539	0.30869426037404	0.30869426037408	0.30869426037408

This example, solved in the thesis (Al-Tamimi's Innovative Second-Class Acceleration Methods for Improving the Results of Integral Values Numerically), gave accuracy using the Simpson method without accelerating to ten decimal places and using accelerations. While our method obtained nine times [6] [5] n=8, we obtained twelve decimal places when Decimal without acceleration when n=12 With acceleration $Æ^{S\$}{}_{\cos^{-1}}$ we obtained fourteen decimal places when n=12 While by accelerating , $Æ^{S\$}{}_{\sin^{-1}}$ $Æ^{S\$}{}_{\tan^{-1}}$ we obtained integer values to thirteen decimal places. Therefore, we expect preference in terms of accuracy for the acceleration referred to in the comment.

3. The integrator $I = \int_7^8 \frac{1}{X^{\frac{1}{2}}+X^{\frac{1}{3}}}\, dx$ dx is continuous in the integration period [7,8] and the formula for the correction limits for Simpson's rule is 3/8 as in the formula No. (1.3).

Table 3: Calculating the Integral $I = \int_7^8 \frac{1}{X^{\frac{1}{2}}+X^{\frac{1}{3}}} dx = 0.191063352613.$ Numerically Using Simpson's Three-Eighth Rule with Acceleration of Trigonometric Functions

N	Values of simpson's 3/8 rule	$Æ^{S\$}{}_{\cos^{-1}}$	$Æ^{S\$}{}_{\sin^{-1}}$	$Æ^{S\$}{}_{\tan^{-1}}$
3	0.19106346005463			
6	0.19106335938938	0.19106335266506	0.1910633526784	0.19106335267831
9	0.19106335395457	0.19106335261573	0.19106335261677	0.19106335261676
12	0.19106335303839	0.19106335261413	0.19106335261433	0.19106335261433
15	0.19106335278785	0.19106335261398	0.19106335261404	0.19106335261404

This example, solved in the thesis (Al-Tamimi's Innovative Second-Class Acceleration Methods for Improving the Results of Integral Values Numerically), gave accuracy using the Simpson method without accelerating to ten decimal places and using accelerations. While our method obtained nine times [6] [5] n=8, we obtained twelve decimal places when Decimal without acceleration when n=12 With acceleration $Æ^{S\$}{}_{\cos^{-1}}$ we get thirteen decimal places when n=15 While by accelerating , $Æ^{S\$}{}_{\sin^{-1}}$ $Æ^{S\$}{}_{\tan^{-1}}$ we obtained integer values to eleven decimal places.

Conclusion

We conclude from the tables 1, 2, 3 above that these acceleration methods work with the same efficiency and give high accuracy in the results with a relatively small number of partial periods.

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