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Every Adjacency Matrix of a Simple Graph is a Soft Matrix

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Abstract

This paper develops a connection between the soft set theory and the graph theory by demonstrating that the adjacency matrix of a simple graph can be modeled as a soft matrix. The authors delve into directed and undirected graphs in which they demonstrate that the adjacency matrix in a simple undirected graph is symmetrical indicating the two-directional nature of the edges. In the case of directed graphs, the adjacency matrix is a symmetric one in case the arc set is a symmetric set, but is non-symmetric otherwise. The paper makes the formal relation between the adjacency matrix and the soft matrix, and it is shown that an adjacency matrix of any simple undirected graph is a symmetric soft matrix. Using mathematical proofs and examples, the authors demonstrate that the adjacency matrix selflessly satisfies the structure of a soft matrix. This work makes a major contribution in the combination of graph theory with the soft set theory since the two are fundamental elements of mathematics. The results provide a fresh view to study graph structures in terms of soft set operations, which can find its way to other fields such as network analysis and soft computing. Also, the paper will show how to model graph relationships using soft matrices, which help to better understand the graph structures in mathematical computations. The aspect of the soft matrices as a tool of graph analysis followed by the authors introduces new opportunities to the further research and implementation, particularly in the areas of data analysis, decision-making, and network theory. This conceptual relationship between soft and adjacency matrices is a significant move in increasing the usefulness of both graph theory and soft set theory.

Keywords: Simple Graph, Undirected & Directed Graph, Adjacency Matrix, Soft Matrix, Symmetric Soft Matrix.

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Introduction

Graph theory and soft set theory are two powerful mathematical concepts that have been extensively used in various fields like computer science, data analysis, and decision-making. Graph theory expresses the relationship between entities in an abstract manner using vertices and edges, whereas soft set theory presented by Molodtsov, (1999) provides a precise way of handling uncertainty and vagueness in data using parameterized structures.

In the last decades, some researchers have been studying the application of soft set theory with other mathematical models. Maji et al., (2003) established simple operations and matrix representations of soft sets, proposing the notion of soft matrices to enable computational analysis. With their operations, Çağman & Enginoğlu, (2010) provided a more useful framework for theoretical investigations in soft set theory by defining the idea of a soft matrix. Subsequent research extended soft matrices to decision-making, fuzzy systems, and data classification applications. On the other hand, adjacency matrices have long been a fundamental algebraic representation of graphs. They describe the connectivity structure of graphs using binary entries that indicate the presence or absence of edges between vertices. Understanding the general application of both frameworks, there are very few works that have directly associated adjacency matrices of graphs with soft matrices of soft sets. The analogy between the binary nature of soft matrices and adjacency matrices is a promising connection between graph-theoretic constructs and soft computing representations (Mukherjee & Mukherjee, 2014).

In this work, we make this connection formally through the proof that: Any adjacency matrix of a simple undirected graph is equivalently represented as a soft matrix. Any adjacency matrix of a directed graph can also be expressed as a soft matrix, although not necessarily symmetric. Besides, we demonstrate that the adjacency matrix (and therefore soft matrix) of an undirected simple graph is symmetric, whereas for a directed graph it is symmetric if and only if the arc set of the graph is symmetric; otherwise, it is non-symmetric (Paramadevan & Sotheeswaran, 2021).

Using theorems, proofs, and examples, this paper establishes a mathematical bridge between graph theory and soft set theory, emphasizing that adjacency matrices naturally fulfill the structure of soft matrices. Such an analogy gives rise to new avenues for the analysis of graphs by soft set operations and for understanding soft matrices in relational network structures.

Preliminaries

Definition 2.1

A graph G = (V, E) consists of a set V of vertices (as nodes) and a set E of edges.

Example 2.1

Consider a simple graph G,

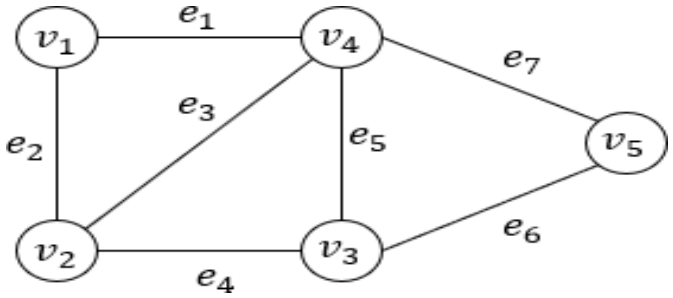


Fig. 1: A Graph G

A simple graph G with vertices set V = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and edge set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ (Fig. 1).

Definition 2.2

The adjacency matrix of a graph G = (V, E) with ordered vertex set $(v_1, v_2, \ldots, v_n)$ is the matrix $A_G = [a_{ij}]_{n\times n}$ with,

$$[a_{ij}] = \begin{cases} 1 & if\ (v_i, v_j) \in E \\ 0 & otherwise \end{cases}$$

Example 2.2

Let the above graph G,

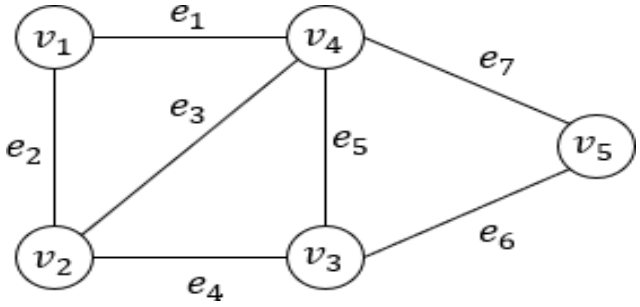


Fig. 2: A Graph G

Here, A simple graph G with vertices set V = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and edge set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\} = \{(v_1, v_2), (v_1, v_4), (v_2, v_4), (v_2, v_3), (v_4, v_3), (v_3, v_5), (v_4, v_5)\}$ (Fig.2).

Hence the adjacency matrix is given by A_G,

$$A_G = [a_{ij}]_{n\times n} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Definition 2.3

Let E be a parameter set associated with U and U be the universal set. $P(U)$ represents the power set of U. F: E→ P(U).is a pair of soft sets (F, E).

Definition 2.4

Let $\mathbb{U}$ be a universe, $\mathbb{E}$ be a set of parameters with related to $\mathbb{U}$ and $\mathcal{A} \subseteq \mathbb{E}$. Let a soft set over $\mathbb{U}$ be $(\tau_A, \mathbb{E})$ can expressed a relation form as a subset $\mathcal{R}_\mathcal{A}$ of $\mathbb{U} \times \mathbb{E}$ where $\mathcal{R}_\mathcal{A} = \{(u, e): e \in \mathcal{A}, u \in \tau_\mathcal{A}(e)\}$.

$\delta_{\mathcal{R}_\mathcal{A}}$ symbolize the characteristic function of $\mathcal{R}_\mathcal{A}$ which is given as $\delta_{\mathcal{R}_\mathcal{A}} : \mathbb{U} \times \mathbb{E} \rightarrow \{0,1\}$, with,

$$\delta_{\mathcal{R}_\mathcal{A}}(u, e) = \begin{cases} 1, & (u, e) \in \mathcal{R}_\mathcal{A} \\ 0, & (u, e) \notin \mathcal{R}_\mathcal{A} \end{cases}$$

if $\mathbb{U} = \{u_1, u_2, \ldots, u_m\}$ and $\mathbb{E} = \{e_1, e_2, \ldots, e_n\}$ then the soft set $(\tau_\mathcal{A}, \mathbb{E})$ can be written in a matrix $[\alpha_{ij}]$ known as $m \times n$ "Soft Matrix" as follows,

$$[\boldsymbol{\alpha_{ij}}]_{\boldsymbol{m\times n}} = \begin{bmatrix} \boldsymbol{\alpha_{11}} & \boldsymbol{\alpha_{12}} & \cdots & \boldsymbol{\alpha_{1n}} \\ \boldsymbol{\alpha_{21}} & \boldsymbol{\alpha_{22}} & \ldots & \boldsymbol{\alpha_{2n}} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{\alpha_{m1}} & \boldsymbol{\alpha_{m2}} & \ldots & \boldsymbol{\alpha_{mn}} \end{bmatrix} \text{ where } \boldsymbol{\alpha_{ij}} = \boldsymbol{\delta_{\mathcal{R}_\mathcal{A}}(u_i, e_i)}$$

A soft matrix of order $m \times n$ is a soft matrix with m rows and n columns.

Example 2.3

Suppose that U = $\{c_1, c_2, c_3, c_4, c_5\}$ is a universal set and E = $\{s_1, s_2, s_3, s_4, s_5\}$ be set of parameters. If A ⊆ E = $\{s_1, s_2, s_4, s_5\}$, the soft set define the parameters as $f_A(s_1) = \{c_1, c_3, c_4\}$, $f_A(s_2) = \{c_2, c_3, c_4, c_5\}$, $f_A(s_4) = \{c_1, c_2, c_4\}$, $f_A(s_5) = \{c_2, c_3, c_4, c_5\}$ then soft set (f_A, E) = $\{(s_1, \{c_1, c_3, c_4\}), (s_2, \{c_2, c_3, c_4, c_5\}), (s_3, \{c_1, c_2, c_4\}), (s_5, \{c_2, c_3, c_4, c_5\})\}$

The soft matrix is given by,

$$[a_{ij}] = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Relation Between Adjacency Matrix and Soft Matrix

This section, we present some theorem based on adjacency matrix and soft matrix with some examples.

Theorem 3.1

Let G = (V, E) be a finite simple undirected graph and Let V = $\{v_1, v_2, \ldots, v_n\}$ be a vertex set. Let universal set U and parameter set E are equal to vertex set.

Define a mapping F: P→ P(U) by F $(v_j) = \{v_i \in V : (v_i, v_j) \in E\}$.

Then the soft matrix M = $[m_{ij}] = \begin{cases} 1 & if\ v_i \in F(v_j) \\ 0 & otherwise \end{cases}$ of the soft set (F, P) is equal to the adjacency matrix A_G of graph G, with respect to ordering of vertices.

Proof

Let V is finite and ordered as $v_1, v_2, \ldots, v_n$ and E ⊆ $V \times V$, We know that the adjacency matrix $A_G = [a_{ij}] \in \{0, 1\}^{n \times n}$ is $[a_{ij}] = \begin{cases} 1 & if\ (v_i, v_j) \in E \\ 0 & otherwise \end{cases}$ and the soft set (F, P) of soft matrix M = $[m_{ij}] = \begin{cases} 1 & if\ v_i \in F(v_j) \\ 0 & otherwise \end{cases}$

To prove $A_G = M$.

Since both A_G and M are n x n matrices, to prove matrix equality, we need to show that every corresponding entry is equal.

For every i, j with $1 \le i, j \le n$. From the definition of soft matrix,$[m_{ij}] = 1 \Leftrightarrow v_i \in F(v_j)$

$\Leftrightarrow$ $(v_i, v_j) \in E$ ∵ the mapping of F

$\Leftrightarrow$ $[a_{ij}] = 1$ ∵ definition of adjacency matrix

∴ $[m_{ij}] = [a_{ij}]$ for every i, j with $1 \le i, j \le n$

(i.e.,) $A_G = M$

Example 3.1

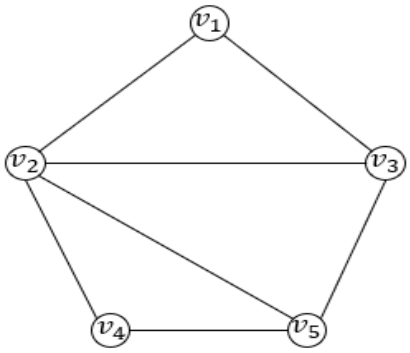


Fig. 3: A Simple Undirected Graph G

Suppose that G = (V, E) be a simple undirected graph. Assume G have the vertices V = $\{v_1, v_2, v_3, v_4, v_5\}$ and edges E = $\{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_2, v_5), (v_5, v_1), (v_2, v_4)$ (Fig.3).

The adjacency matrix is given as $A_G = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

We define F$(v_j) = \{v_i \in V : (v_i, v_j) \in E\}$

From the edge set, we construct the soft matrix as,

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$\therefore A_G = M$

Adjacency matrix of given graph G is a soft matrix.

Fig. 4 depicts the Friendship graph f10.

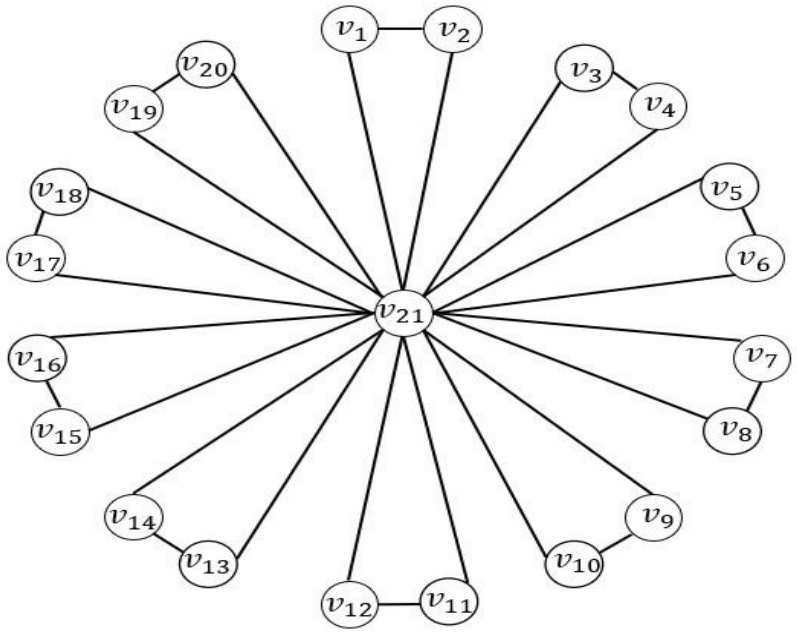


Fig. 4: Friendship Graph F10

The adjacency matrix A_G is given as,

$$A_G = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

We define F(v_j) = $\{v_i \in V : (v_i, v_j) \in E\}$, From the edge set, we construct the soft matrix as

$$M = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\therefore A_G = M$$

Theorem 3.2

Consider G = (V, E) be a finite simple directed graph and assume V = {$v1, v2, \ldots, vn$} be a vertex set. Let universal set U and parameter set E are equal to vertex set. Define a mapping F: P→ P(U) by F(vj) = $\{vi \in V : (vi, vj) \in E\}$. Let the soft matrix M of the soft set (F, P) is defined as $M = [m_{ij}] = \begin{cases} 1 & if\ v_i \in F(v_j) \\ 0 & otherwise \end{cases}$ is equal to the adjacency matrix A_G of graph G , with respect to ordering of vertices.

Proof

Let V is finite and ordered as $v_1, v_2, \ldots, v_n$ and E ⊆ $V \times V$

To prove: $A_G = M$, the adjacency matrix $A_G = [a_{ij}] \in \{0, 1\}^{n \times n}$ is $[a_{ij}] = \begin{cases} 1 & if\ (v_i, v_j) \in E \\ 0 & otherwise \end{cases}$

Since both A_G *and* M are n x n matrices, to prove matrix equality, we need to show that every corresponding entry is equal.

For every i, j with $1 \le i, j \le n$. From the definition of soft matrix

$[m_{ij}] = 1 \Leftrightarrow v_i \in F(v_j)$

$\Leftrightarrow (v_i, v_j) \in E$ ∵ the mapping of F

$\Leftrightarrow [a_{ij}] = 1$ ∵ definition of adjacency matrix

$\therefore [m_{ij}] = [a_{ij}]$ for every i, j with $1 \le i, j \le n$

(i.e.,) $A_G = M$.

Example 3.2

Suppose that G = (V, E) be a simple directed graph. Let G have the vertices V = $\{v_1, v_2, v_3, v_4, v_5\}$ and edges E = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}\}$ (Fig. 5).

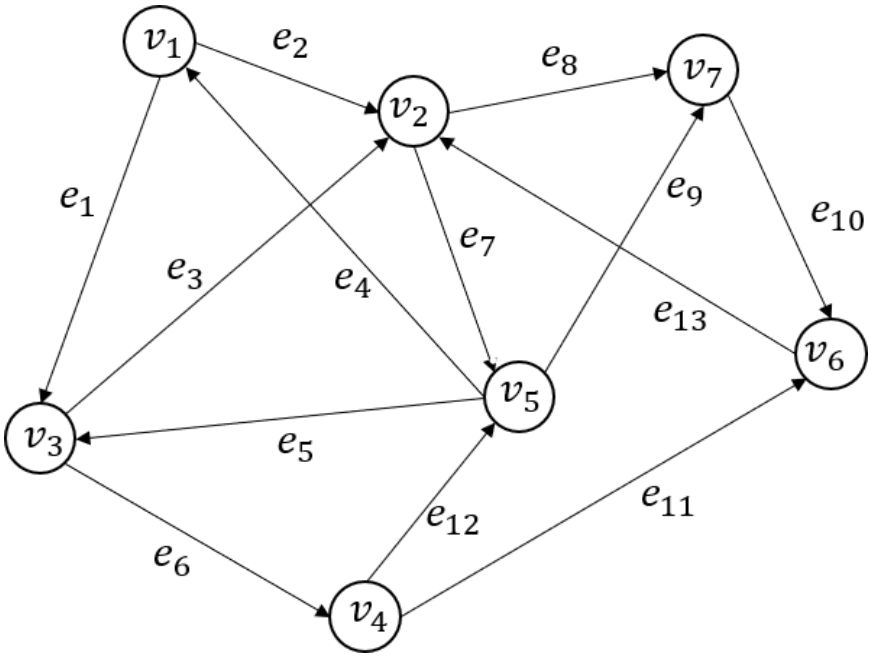


Fig. 5: A Simple Directed Graph G

The adjacency matrix A_G is given as,

$$A_G = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

We define $F(v_j) = \{v_i \in V : (v_i, v_j) \in E\}$

From the edge set, we construct the soft matrix as,

$$M = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Fig. 6 shows the Complete graph K9 The adjacency matrix A_G is given as $\therefore A_G = M$

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

We define $F(v_j) = \{v_i \in V : (v_i, v_j) \in E\}$,

From the edge set, we construct the soft matrix as $\therefore AG = M$.

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

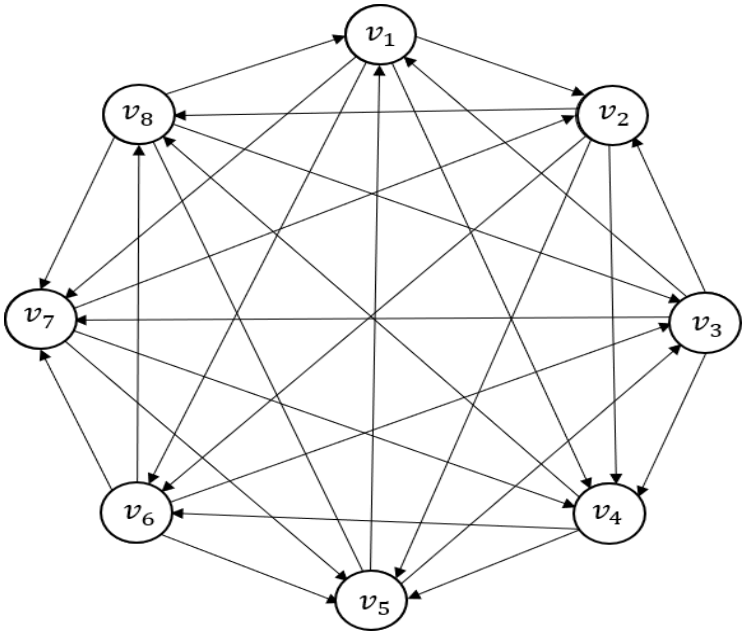


Fig. 6: Complete Graph K9

Theorem 3.3

All adjacency matrices of a simple undirected graph is a symmetric soft matrix.

Proof

Consider G = (V, E) be a finite simple undirected graph and Let V = $\{v_1, v_2, \dots, v_n\}$ be a vertex set. Let universal set U and parameter set E are equal to vertex set.

A mapping is defined as F: P→ P(U) by F(v_j) = $\{v_i \in V : (v_i, v_j) \in E\}$.

Then the soft matrix M = $[m_{ij}] = \begin{cases} 1 & if\ v_i \in F(v_j) \\ 0 & otherwise \end{cases}$ of the soft set (F, P).

We know that "every adjacency matrix of a graph is a soft matrix", Here a_{ij} = 1 exactly when $\{v_i, v_j\} \in E_G$ because G is undirected (i.e.) an edge $\{v_i, v_j\}$ is the same unordered pair as $\{v_j, v_i\}$.

Hence for every i, j.

Theorem 3.4

a_{ij} = 1 ⇔ $\{v_i, v_j\} \in E_G$

⇔ $\{v_j, v_i\} \in E_G$

⇔ a_{ji} = 1

∴ $a_{ij} = a_{ji}$.

All adjacency matrix of a simple directed graph is a symmetric soft matrix if and only if the graph's arc set is symmetric; otherwise, it is non-symmetric.

Proof

Suppose that G = (V, E) be a finite simple undirected graph and assume V = $\{v_1, v_2, \ldots, v_n\}$ be a vertex set. Let universal set U and parameter set E are equal to vertex set.

Define a mapping F: P→ P(U) by F(v_j) = $\{v_i \in V : (v_i, v_j) \in E\}$.

The adjacency matrix (or Soft Matrix) is $A_G = [a_{ij}] \in \{0, 1\}^{n \times n}$ is $[a_{ij}] = \begin{cases} 1 & if\ (v_i, v_j) \in E \\ 0 & otherwise \end{cases}$

(⇒) If A_G is Symmetric, then E_G is Symmetric

Suppose A_G is symmetric, then $a_{ij} = a_{ji}$ for all i, j. If $(v_i, v_j) \in E_G$, then by definition $a_{ij} = 1$.

Since A_G is symmetric, $a_{ji} = 1$, which implies $\{v_j, v_i\} \in E_G$

Hence, for every directed edge $\{v_i, v_j\}$, its reverse $\{v_j, v_i\}$ also exists.

∴ The arc set E_G is symmetric.

(⇐) If E_G is Symmetric, then A_G is Symmetric

Suppose the arc set E_G is symmetric, (i.e.) $\{v_i, v_j\} \in E_G \Leftrightarrow \{v_j, v_i\} \in E_G$

Then, for every i, j

$a_{ij} = 1 \Leftrightarrow \{v_i, v_j\} \in E_G \Leftrightarrow \{v_j, v_i\} \in E_G \Leftrightarrow a_{ji} = 1$

∴ $a_{ij} = a_{ji}$ for all i, j Hence, A is symmetric.

Therefore, adjacency matrix is symmetric soft matrix if and only if arc set E_G is symmetric, otherwise it is non-symmetric.

Example 3.3

Suppose that G = (V, E) be a simple directed graph. Consider G have the vertices V = $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}\}$ and edges E = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}\}$ (Fig. 7).

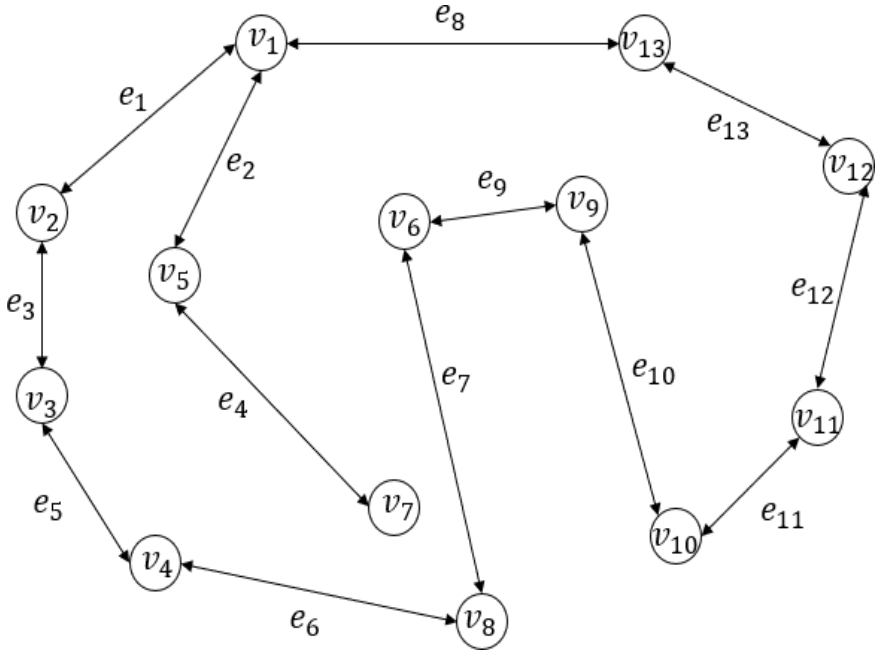


Fig. 7: A Simple Directed Graph

The adjacency matrix AG or Soft Matrix M is given as $A_G =$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here, $a_{12} = a_{21}. \therefore a_{ij} = a_{ji}.$

The adjacency matrix of a simple directed graph is a symmetric soft matrix.

Example 3.4

Consider G = (V, E) be a simple directed graph. Let G have the vertices V = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and edges E = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ (Fig. 8).

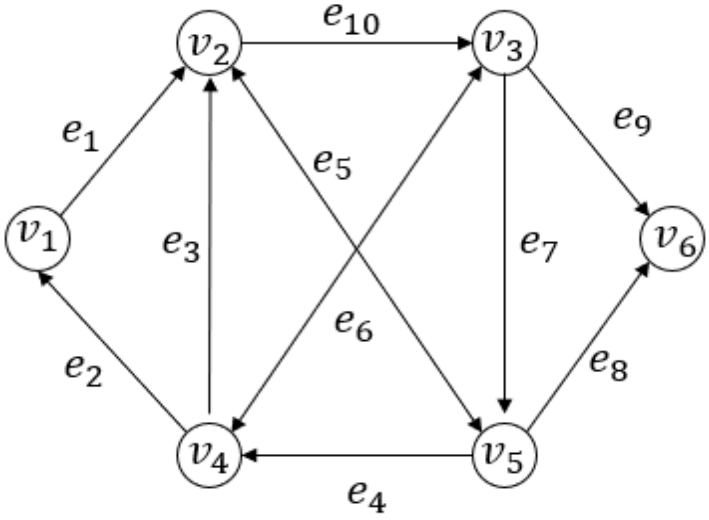


Fig. 8: A Simple Directed Graph

The adjacency matrix A_G or Soft Matrix M is given as,

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, $a_{12} \neq a_{21}, \therefore a_{ij} \neq a_{ji}.$

The adjacency matrix of a simple directed graph is a non-symmetric soft matrix.

Conclusion

This paper shows the relation between graph theory and soft set theory by demonstrating that each adjacency matrix of a simple undirected or directed graph can be expressed as a soft matrix. It also illustrates how undirected graphs generate symmetric soft matrices and directed graphs are symmetric only in the case of symmetric arc sets. These findings lay the ground for more applications in network analysis and soft computing.

References

Mukherjee, C., & Mukherjee, G. (2014). Role of adjacency matrix in graph theory. *IOSR Journal of Computer Engineering*, *16*(2), 58-63.

Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & mathematics with applications*, *45*(4-5), 555-562. https://doi.org/10.1016/S0898-1221(03)00016-6

Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, *37*(4-5), 19-31. https://doi.org/10.1016/S0898-1221(99)00056-5

Çağman, N., & Enginoğlu, S. (2010). Soft matrix theory and its decision making. *Computers & mathematics with applications*, *59*(10), 3308-3314. https://doi.org/10.1016/j.camwa.2010.03.015

Paramadevan, P., & Sotheeswaran, S. (2021). Properties of adjacency matrix of a graph and it's construction. *Journal of Science*, *12*(1).