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Fuzzy Truncated Life Test Plans for the Type-II Exponentiated Log-Logistic Distribution with Application to Cardiac Stent Reliability

H. Shubhajyothi [a*], Dr.K. Rebecca Jebaseeli Edna [b], Dr.V. Jemmy Joyce [c], P. Anitha [d]

[a*] *Assistant Professor, Department of Mathematics, Sri Krishna College of Engineering & Technology, Kuniamuthur, Tamil Nadu, India; Research Scholar, Department of Mathematics, Karunya Institute of Technology and Sciences, Coimbatore, Tamil Nadu, India.*
E-mail: apurvaajyothi@gmail.com, Orcid: https://orcid.org/0009-0001-2030-5464
[b] *Associate Professor, Department of Mathematics, Karunya Institute of Technology and Sciences, Coimbatore, Tamil Nadu, India. E-mail: krjedna@gmail.com, Orcid: https://orcid.org/0000-0001-7656-9832*
[c] *Assistant Professor, Department of Mathematics, Karunya Institute of Technology and Sciences, Coimbatore, Tamil Nadu, India. E-mail: vjemmyjoyce@gmail.com, Orcid: https://orcid.org/0000-0003-4343-4135*
[d] *Assistant Professor, Department of Applied Science and Humanities, Nehru College of Engineering and Research Centre, Pampady, Thrissur, Kerala, India; Research Scholar, Department of Mathematics, Karunya Institute of Technology and Sciences, Coimbatore, Tamil Nadu, India.*
E-mail: anitha.maths@ncerc.ac.in, Orcid: https://orcid.org/0009-0008-0467-3356

Abstract

Classical truncated life test acceptance sampling plans assume exact knowledge of lifetime distribution parameters - an assumption rarely satisfied in high-stakes reliability applications such as drug-eluting cardiac stent qualification, where scale parameter estimates carry inherent epistemic uncertainty from limited pilot data or expert judgment. This paper addresses that gap by developing a fuzzy acceptance sampling plan for the Type-II Exponentiated Log-Logistic (TII-ELL) distribution under truncated life testing, generalizing the classical crisp framework to a fuzzy environment. The scale parameter corresponding to the specified minimal mean life is modeled as a triangular fuzzy number. Using the λ-cut representation and Zadeh's extension principle, closed-form interval expressions are derived for the fuzzy failure probability and fuzzy operating characteristic (OC) function. A key theoretical result, proved as Theorem 5.1, establishes that the TII-ELL failure probability is strictly monotone decreasing in the scale parameter, enabling efficient endpoint-based λ-cut propagation without exhaustive search. Fuzzy interval formulations of consumer's and producer's risk are derived, and a systematic iterative algorithm determines the jointly optimal fuzzy sample size and acceptance number satisfying dual risk constraints across all λ-cut levels. Applied to drug-eluting cardiac stent reliability data with truncation time t= 36 months and plan (n=22, c=2 & n=23, c=2), the fuzzy OC band confirms lot acceptance probability below 0.11% even under optimistic parameter assumptions - demonstrating that the proposed framework provides stronger, more transparent consumer protection than its crisp counterpart under realistic parameter uncertainty. The framework is theoretically consistent and reduces to the classical crisp plan as fuzziness vanishes, confirming its validity as a mathematically sound and practically robust alternative to deterministic reliability-based quality control.

Keywords: Fuzzy Acceptance Sampling, Truncated Life Test, TII-ELL Distribution, Fuzzy Operating Characteristic Curve, Consumer's Risk, Reliability Analysis, Medical Device Reliability.

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Introduction

Abbreviated life test acceptance sampling schemes are widely applied in reliability engineering for evaluating product performance when obtaining complete lifetime observations is impractical because of extended testing durations, high operational costs, or destructive evaluation procedures. Such plans were first considered for the lifetime distribution that follows the exponential model and were subsequently considered for several other lifetime distribution models. This approach consists of performing tests on a fixed number of items and stopping the experiment when a specified truncation time is reached. The number of acceptances denotes the maximum number of defects in a lot that is acceptable. This scheme considerably decreases the testing time while offering statistically reliable safeguards to both the producer and the consumer. Various studies have confirmed the success of truncated lifetime test designs in different lifetime distributions, making them applicable in reliability and quality control.

Conventional truncated life test plans are generally formulated assuming that the underlying lifetime distribution and its corresponding parameters are known with certainty. In real-life reliability testing, the parameters of the model in question are commonly estimated from scarce data obtained during pilot tests, accelerated tests, or subjective evaluations related to biomedical equipment, aircraft, and reliable parts used in industry. Estimation of the parameters is necessarily connected with epistemic uncertainty. Considering the estimates as independent crisp values can lead to distorted interpretation of producer and consumer risks.

The TII-ELL distribution represents a generalized form of the conventional log-logistic distribution. This adaptable lifetime distribution is capable of representing various hazard rate patterns, including increasing, decreasing, and bathtub-shaped failure behaviors. It is used more and more for dependability modeling of engineering and biological systems since it is flexible and has a manageable mathematical structure. Rao et al., (2009) introduced conventional acceptance sampling procedures for the TII-ELL lifetime model using a truncated life test framework. Minimum sample sizes and operating characteristic values were obtained for different confidence levels and acceptance numbers for a deterministic setting of parameters.

Fuzzy set theory offers a mathematical framework for handling uncertainty by enabling the representation of vague, incomplete, or imprecise information using flexible structures. Further developments have expanded its theoretical framework for representing imprecise and uncertain information. Membership functions, represented through fuzzy numbers, are used instead of assigning a single deterministic value to uncertain parameters. Conversion of probabilistic quantities into interval-valued or graded measures can be achieved using the extension principle and λ-cut representation (Turanoğlu et al., 2012). This approach is most useful for reliability situations where the sample sizes are small, expert opinion is used, and the language used contains criteria such as "approximately equal to" a target mean life, which are inherently uncertain. The majority of the literature is based on attribute sampling plans or complete life testing, but fuzzy acceptance sampling and fuzzy reliability analysis have been developed for many classical lifetime models (Bhattacharyya, 2023; Khan et al., 2022).

The main objective of this study is to extend the classical framework proposed by Rao et al., (2009) into a fuzzy setting by developing a fuzzy acceptance sampling plan based on truncated life testing for products following the TII-ELL life distributions. The scale parameter associated with the specified minimum mean life is represented using a triangular fuzzy number. The extension principle, λ-cut analysis, and monotonicity properties of the failure probability are applied to derive explicit interval expressions for the fuzzy operational characteristic functions and the fuzzy failure probabilities. A systematic method for calculating the fuzzy sample size and building fuzzy consumer and producer hazards is proposed. The utility and robustness of the proposed framework is demonstrated by the example of medical equipment dependability.

The key contributions of this study can be summarized as follows:

- The failure probability in a truncated life test is converted from a crisp scalar to an interval-valued fuzzy quantity by using the monotonicity of the TII-ELL CDF and modeling the scale parameter as a triangular fuzzy number.
- The fuzzy OC function is expressed in closed form λ-cut (interval) expressions, yielding a fuzzy OC band. This band measures the probability of lot acceptance under pessimistic, most probable, and optimistic parameters.
- By adopting fuzzy formulations of the risks encountered by both producers and consumers as interval numbers, decision-makers are given a range of risk levels rather than a single, perhaps misleading, crisp figure.
- In this paper, a structured iterative method is proposed to determine the fuzzy sample size $\tilde{n} = (n_L, n_M, n_U)$ for the three components of the triangle fuzzy scale parameter. As the fuzziness disappears, the process goes back to the classical plan, which guarantees theoretical consistency.

- The framework has been illustrated on a reliability challenge with drug-eluting cardiac stents, where the fuzzy sampling plan has been shown to be reliable for consumer protection (lot acceptance probability < 0.11%) under optimistic parameter assumptions.

The remainder of the paper is organized as follows. Section II presents the review of relevant literature. Section III describes the TII-ELL distribution along with its fundamental definitions. Section IV explains the development of the fuzzy acceptance sampling framework. Section V provides the mathematical formulation and theoretical foundation. Section VI discusses the numerical illustration, and Section VII concludes the paper with potential directions for future research.

Acceptance sampling for high-reliability components such as drug-eluting cardiac stents requires lifetime models flexible enough to capture diverse hazard shapes, together with decision procedures that remain valid under the inherent ambiguity of borderline failure classification. This paper addresses both needs by developing a fuzzy double sampling acceptance plan based on TII-ELL distribution.

The drug-eluting stent example is representative because it involves small, costly destructive life tests, severe failure consequences, and borderline failure classification (partial strut fracture, marginal coating loss) near the inspection boundary - conditions where fuzzy uncertainty is genuinely present rather than assumed. The fuzzy plan improves decision-making by retaining this borderline information instead of forcing a binary pass/fail cutoff, producing a smoother operating characteristic curve, reducing misclassification of ambiguous units, and yielding a quantifiable indeterminate decision band rather than a single crisp threshold - giving engineers more defensible, risk-calibrated lot-acceptance decisions for high-reliability implantable devices.

Literature Review

Truncated Life Test Acceptance Sampling Plans

Acceptance sampling based on truncated life tests has long served as a practical compromise between the cost of full destructive testing and the need for statistically defensible lot-disposition decisions. The conventional framework considers a fixed truncation time $t$and determines the design parameters, including the sample size $n$and acceptance number c, such that the predefined producer's and consumer's risk requirements are satisfied at specific quality levels. This framework has been applied and expanded to various lifetime distributions over recent years. Hamurkaroğlu et al., (2020) introduced single and double acceptance sampling plans for the compound Weibull–exponential distribution, whereas Tripathi et al., (2021) developed double and group sampling procedures for the inverse log-logistic distribution. Saha et al., (2021) generalized the approach for the transmuted Rayleigh distribution, and Singh et al., (2019) investigated acceptance sampling plans for the generalized Pareto distribution. Al-Omari et al., (2021) proposed sampling plans based on the two-parameter Quasi Shanker distribution while ensuring mean life requirements and demonstrated their applicability using manufacturing data. Further developments include acceptance sampling strategies for the Tsallis q-exponential distribution (Al-Nasser & Obeidat, 2020; Qomi & Fernández, 2024), the three-parameter inverted Topp–Leone distribution (Nassr et al., 2022), and percentile-based sampling plans for the new Weibull–Pareto distribution with applications to carbon fiber breaking-stress data (Shrahili et al., 2021). Collectively, these studies establish the standard two-point OC-curve design methodology this paper builds upon, but each is tied to a specific, often less flexible, parametric lifetime model.

Extensions to Generalized and Exponentiated Lifetime Distributions

A parallel and increasingly active strand of research has focused on extending the underlying lifetime distributions themselves to accommodate more complex hazard behavior - monotone, unimodal, or near-bathtub shaped - that simpler models such as the exponential or basic Weibull cannot capture. Almarashi et al., (2021) developed a group acceptance sampling plan using the Marshall–Olkin Kumaraswamy exponential distribution, whereas Ameeq et al., (2023) presented a comparable sampling strategy based on the alpha power transformation inverted Perks distribution. Naz et al., (2023) proposed a transmuted modified power-generated distribution family and validated its effectiveness through insurance and reliability applications, further highlighting the growing importance of flexible exponentiated and transmuted distribution classes in practical statistical modeling. From a distributional perspective, Alfaer et al., (2021) investigated the inferential and actuarial characteristics, Kariuki et al., (2024) studied the extended log-logistic distribution's estimation behavior across multiple methods, The importance of selecting an appropriate alternative to the Weibull model for reliability data has also been underscored in wind-speed and engineering applications (Akgül et al., 2016), further motivating the adoption of more flexible families such as the TII-ELL distribution in this paper and Kamnge & Chacko, (2025) introduced a half-logistic exponentiated inverse Rayleigh distribution explicitly designed to capture both monotonic and non-monotonic hazard rates. Ihtisham et al., (2024) further proposed a Pareto exponentiated log-logistic family applied to COVID-19 mortality data, demonstrating the broad applicability of exponentiation mechanisms beyond reliability engineering. Despite this growth in flexible distributional families, sampling-plan design for the specific TII-ELL distribution - and, critically, formal proof of the monotonicity property required to justify its operating characteristic function - has not previously appeared in the literature.

Fuzzy and Neutrosophic Acceptance Sampling

Because classical acceptance sampling assumes crisp, unambiguous classification of each tested unit as failed or surviving, a substantial literature has emerged to address the reality that such classification is often imprecise. Aslam, (2018) introduced sampling plan development for the exponential distribution using the neutrosophic statistical interval approach and subsequently expanded this concept to general attribute sampling plans (Aslam, 2019) and Weibull-based group sampling plans considering uncertainty (Aslam et al., 2019). Işık & Kaya, (2021) proposed acceptance sampling procedures based on intuitionistic fuzzy linguistic terms to incorporate both membership and non-membership information in ambiguous classification situations, followed by the development of a hesitant fuzzy linguistic term set framework for similar applications (Işık & Kaya, 2022). Bhattacharyya, (2023) presented a comprehensive analysis of single, double, chain, and sequential sampling schemes under fuzzy parameters by employing fuzzy Poisson distributions and hypothesis testing techniques. While this body of work has substantially advanced the theoretical machinery for incorporating fuzziness into sampling-plan design, the prevailing approach treats the fraction defective or the plan parameters themselves as fuzzy inputs, rather than deriving fuzzy failure probabilities analytically from a fuzzy scale parameter through the monotone structure of an underlying lifetime CDF the propagation mechanism developed in this paper.

Reliability of Drug-Eluting Cardiac Stents

Drug-eluting cardiac stents represent class of high-consequence, low-tolerance medical devices for which reliability assurance is both clinically critical and methodologically demanding. Epidemiological and clinical evidence indicates that coronary stent fracture occurs in non-trivial proportion of implanted devices: a Comprehensive literature review and meta-analysis by Chen et al., (2022) estimated stent fracture incidence at approximately 5.5% among patients and 4.9% among stents, with incidence rates varying significantly by device type. Comparative safety evidence for drug-eluting versus bare-metal stents in other vascular contexts has similarly highlighted the importance of rigorous reliability assessment (Zhang et al., 2024). However, this literature is predominantly clinical and epidemiological in nature; it does not employ statistical acceptance sampling methodology, nor does it consider the role of measurement or classification ambiguity in determining device failure during life testing - a gap directly relevant to the fuzzy sampling framework proposed here.

Research Gap and Motivation

Taken together, the reviewed literature reveals that truncated life test sampling plans have been formulated for an expanding catalogue of lifetime distributions, that increasingly flexible exponentiated and generalized families have been proposed to model complex hazard behavior, that fuzzy and neutrosophic extensions have addressed classification ambiguity in sampling-plan design, and that drug-eluting stent reliability has been documented from a clinical perspective - but no prior study has unified these strands. Specifically, no sampling plan has been developed for the TII-ELL distribution; the monotonicity of its failure probability in the scale parameter has not been formally established; existing fuzzy sampling frameworks do not derive fuzzy failure probabilities via monotone propagation from a fuzzy scale parameter; and no fuzzy sampling plan has been applied to drug-eluting stent reliability data. This paper addresses each of these gaps directly relevant to the fuzzy sampling framework proposed here.

Table 1 summarizes the symbols, parameters, and mathematical notations used throughout the study, including distribution parameters, acceptance sampling characteristics, fuzzy scale parameters, risk measures, α-cut representations, and operating characteristic functions.

Table 1: List of Notations

Symbol	Definition
α	TII-ELL exponentiation shape parameter
β	TII-ELL secondary shape parameter
σ	TII-ELL scale parameter
t	Fixed truncation time
X	Lifetime random variable
$G(t;\alpha,\beta,\sigma)$	TII-ELL CDF/failure probability
$L(p)$	OC function
n	Size of the Sample
c	Acceptance number
Y	Random number of failures, y=observed value
α_p	Risk of Producer
β_c	Risk of Consumer
$\tilde{\sigma}=(\sigma_L,\sigma_M,\sigma_U)$	Triangular fuzzy scale parameter
$\lambda\in[0,1]$	α-cut confidence level
$(\tilde{\sigma})_\lambda=[\sigma_\lambda^-,\sigma_\lambda^+]$	$\lambda-cut$of $\tilde{\sigma}$
σ_0,σ_1	Good/bad quality scale levels ($\sigma_1<\sigma_0$)
α_p	Producer's risk
$1-\alpha_p$	Producer's confidence
$Z_{1-\alpha_p}$	Critical normal value
β_p	Consumer's risk
$1-\beta_p$	Consumer's confidence
$Z_{1-\beta_c}$	Critical normal value
$\tilde{\sigma}_0=(\sigma_{0L},\sigma_{0M},\sigma_{0_U})$	Fuzzy good quality scale parameter
$\tilde{\sigma}_1=(\sigma_{1L},\sigma_{1M},\sigma_{1_U})$	Fuzzy bad quality scale parameter
$(\tilde{\sigma}_0)_\lambda=\left[\sigma_{0_\lambda^-},\sigma_{0\lambda}^+\right]$	$\lambda-cut$of $\tilde{\sigma}_0$
$(\tilde{\sigma}_1)_\lambda=\left[\sigma_{1_\lambda^-},\sigma_{1\lambda}^+\right]$	$\lambda-cut$of $\tilde{\sigma}_1$
$(\tilde{p}_0)_\lambda=\left[p_{0_\lambda^-},p_{0\lambda}^+\right]$	$\lambda-cut$of fuzzy failure probability of $\tilde{\sigma}_0$
$(\tilde{p}_1)_\lambda=\left[p_{1_\lambda^-},p_{1\lambda}^+\right]$	$\lambda-cut$of fuzzy failure probability of $\tilde{\sigma}_1$
$S_p(\sigma)$	Local sensitivity coefficient = $\left\lvert\frac{dp}{d\sigma}\right\rvert$
Δp	Propagated half-width of fuzzy failure probability (Linearized propagation)
$\Delta\sigma$	Half-width of fuzzy scale parameter=$\sigma_U-\sigma_M or\ \sigma_M-\sigma_L$
$[n^-,n^+],[c^-,c^+]$	Fuzzy n, c bounds
$L(p)$	Operating Characteristic function
$L(\tilde{p})$	Fuzzy Operating Characteristic function

Preliminaries

Definition -Type II Exponentiated Log-Logistic Distribution

Assume that the lifetime variable $X$of a product follows the TII-ELL distribution with shape parameters $\alpha>0$, $\beta>0$, and scale parameter $\sigma>0$. The corresponding cumulative distribution function (CDF) is expressed in equation (1).

$$G(t;\alpha,\beta,\sigma)=1-\left[1+\left({}^{t}/_{\sigma}\right)^{\beta}\right]^{-\alpha};t>0\ldots\ldots\ldots\ldots\ldots(1)$$

The corresponding probability density function (PDF) is stated in equation (2).

$$g(t;\alpha,\beta,\sigma)=\frac{\alpha\beta}{\sigma}\left(\frac{t}{\sigma}\right)^{\beta-1}\left[1+\left(\frac{t}{\sigma}\right)^{\beta}\right]^{-(\alpha+1)},t>0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

This distribution is a flexible generalization of the standard log-logistic model and can accommodate a variety of hazard rate shapes, making it suitable for modeling diverse lifetime behaviors encountered in reliability analysis when $\alpha = 1$,the distribution reduces to the classical log-logistic distribution.

Definition -Truncated Life Test and Failure Probability

In a truncated life test setting, $n$units are simultaneously subjected to testing, and the experiment is stopped at a predetermined truncation time t. A failure is considered to occur when a test unit does not survive beyond the specified time t. The probability of failure before the truncation time is represented as $p = G(t)$, and the corresponding crisp probability is defined in equation (3).

$$p = 1 - \left[1 + \left({t}/{\sigma}\right)^{\beta}\right]^{-\alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Definition-Operating Characteristic Function

The operating characteristic (OC) function, which represents the likelihood of accepting a lot, is defined as equation (4):

$$L(p) = P(Y \leq c) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \ldots\ldots (4)$$

Fuzzy Acceptance Sampling Methodology

Definition

A Triangular Fuzzy Number (TFN) is one of the simplest and most widely used representations of uncertainty in fuzzy set theory. It is particularly useful when a parameter is described linguistically as "approximately equal to" some value. A triangular fuzzy number $\tilde{\sigma}$is defined by an ordered triplet $\tilde{\sigma} = (\sigma_L, \sigma_M, \sigma_U)$,where:$\sigma_L$ = lower bound (pessimistic value), σ_M= modal value (most plausible value) σ_U = upper bound (optimistic value). The membership function of $\tilde{\sigma}$ is given by equation (5):

$$\frac{z - \sigma_l}{\sigma_M - \sigma_L}, \sigma_L \leq z \leq \sigma_M \mu_{\tilde{\sigma}}(z) = \frac{\sigma_U - z}{\sigma_U - \sigma_M}, \sigma_M < z \leq \sigma_U \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$0, otherwise$

The value $\mu_{\tilde{\sigma}}(z) \in [0,1]$ represents the degree of membership of z in the fuzzy set z is a member. $\tilde{\sigma}$: values near σ_M have membership close to 1(high plausibility), while values near $\sigma_L$or σ_U have the membership close to 0 (low plausibility). In this paper, the scale parameter σ of the TII-ELL distribution is modeled as a triangular fuzzy number $\tilde{\sigma} = (\sigma_L, \sigma_M, \sigma_U)$, where $\sigma_M$is the nominal engineer's estimate of the characteristic life, and σ_L, σ_U represent the pessimistic and optimistic respectively, reflecting the epistemic uncertainty arising from limited pilot data or expert judgment.

Definition λ-Cut of a Triangular Fuzzy Number: Let $\tilde{\sigma} = (\sigma_L, \sigma_M, \sigma_U)$, be a triangular fuzzy number with $\sigma_L \leq \sigma_M \leq \sigma_U$. The λ-cut of $\tilde{\sigma}$ given by equation (6) for $\lambda \in [0,1]$.

$$\tilde{\sigma}_\lambda = [\sigma_L + \lambda(\sigma_M - \sigma_L), \sigma_U - \lambda(\sigma_U - \sigma_M)] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Fuzzy Acceptance Sampling Plan: This section describes a sequential procedure for developing fuzzy acceptance sampling plans under truncated life testing, assuming that the product lifetime follows the TII-ELL distribution. The procedure is exhibited using a flowchart in fig. 1.

Algorithm: Construction of Fuzzy Acceptance Sampling Plan

1. Specify the lifetime model: Consider a product lifetime variable Xthat follows the TII-ELL distribution, where the shape parametersα and β, along with the scale parameter σ.

2. Define the fuzzy scale parameter: Let the specified minimum scale parameter corresponding to the required mean life be represented by a fuzzy number $\tilde{\sigma}_0 = (\sigma_{0L}, \sigma_{0M}, \sigma_{0M})$ $\sigma_{0L}, \sigma_{0M}, \sigma_{0M}$denote the lower, modal, and upper bounds, respectively.

Obtain $\lambda - cut$ of $\tilde{\sigma}_0$ as equation (7):

$$(\tilde{\sigma}_0)_\lambda = [\sigma_{\lambda L}, \sigma_{\lambda M}], \lambda \in [0,1] \ldots\ldots\ldots\ldots\ldots (7)$$

3. Fix Test Parameters: Specify the Truncation time t, Acceptance number c, Fuzzy consumer's confidence level $\tilde{p}\,*$(if applicable) Desired $\lambda-$level resolution.

4. Compute fuzzy failure probability: For each $\boldsymbol{\lambda - cut}$**,** compute the interval-valued failure probability using the monotonicity equation (8):

$$G(t;\alpha,\beta,\sigma);\ (\tilde{p})_\lambda = [G(t;\alpha,\beta,\sigma_{\lambda U}), G(t;\alpha,\beta,\sigma_{\lambda L})]\ldots\ldots(8)$$

5. Determine fuzzy operating characteristic function: For a given sample size n, compute the $\lambda - cut$ of the fuzzy operating characteristic function using equation (9):

$$(\tilde{L})_\lambda = [L(p_{\lambda L}), L(p_{\lambda U})]\ \ldots\ldots\ldots.(9)$$

Where $L(p) = \sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i}$

6. Consumer's Risk Verification: For each $\lambda-$level verify the fuzzy consumer's risk Condition using equation (10):

$$sup(\tilde{L})_\lambda \le inf(1-\tilde{p}^*)_\lambda\ldots\ldots\ldots\ldots(10)$$

If the condition is not satisfied increment n and repeat steps: 5-6.

7. Determine Fuzzy Sample Size: The smallest integer n satisfying the above condition for all $\lambda \in [0,1]$ constitutes the fuzzy sample size, denoted by equation (11).

$$\tilde{n} = (n_L, n_M, n_U)\ldots\ldots.(11)$$

Corresponding to pessimistic, most likely and optimistic decision scenarios.

8. Fuzzy Acceptance Decision Rule: A truncated life test is performed on $n$units, and the observed number of failures y occurring before the specified truncation time $t$is represented in equation (12).

If $Y \le c$,the lot is accepted with degree

$$\mu_{Accept} = sup\{\lambda: (\tilde{L})_\lambda \ge (\tilde{p}^*)_\lambda\ldots..(12)$$

If $Y > c$,the lot is rejected.

9. Producer's Risk Evaluation: For the specified true scale parameter $\sigma > \sigma_0$, compute the fuzzy producer's risk using the approach as equation (13):

$$\tilde{\beta} = 1 - \tilde{L}\big(G(t;\alpha,\beta,\sigma)\big)\ldots\ldots\ldots\ldots..(13)$$

Note: When the fuzzy scale parameter $\tilde{\sigma}_0$becomes a single deterministic value, the proposed procedure converges to the conventional truncated life test acceptance sampling plan, demonstrating the validity and consistency of the fuzzy formulation.

Mathematical Foundations of the Fuzzy Model

This section establishes the theoretical justification for the fuzzy acceptance sampling plan developed for the TII-ELL distribution. In particular, the monotonicity of the failure probability with respect to the scale parameter is proved, and the λ-cut interval form of the fuzzy operating characteristic function is derived.

Theorem Monotonicity of Failure Probability

Let$X$~TII-ELL distribution with shape parameters $\alpha > 0, \beta > 0$and scale parameter $\sigma > 0$.Let the failure probability under a truncated life test at time t be $p(\sigma) = G(t;\alpha,\beta,\sigma)$.

Therefore, for fixed values of t, α, and β, the failure probability decreases monotonically with an increase in the scale parameter σ as shown in equation (14) and (15).

$$G(t;\alpha,\beta,\sigma) = 1 - \left[1 + \left({}^{t}/_{\sigma}\right)^{\beta}\right]^{-\alpha};\ t > 0\ldots\ldots\ldots\ldots.(14)$$

$$Let\quad u(\sigma) = 1 + \left(\frac{t}{\sigma}\right)^{\beta}\ \&p(\sigma) = 1 - u(\sigma)^{-\alpha}\ldots\ldots\ldots..(15)$$

Steps: 1) Then differentiating equation with respect to equation (16).

$$\sigma \Rightarrow \frac{dp}{d\sigma} = \alpha u(\sigma)^{-\alpha-1}\frac{du}{d\sigma}\ldots\ldots\ldots\ldots(16)$$

2) Differentiate u as shown in equation (17) and (18).

$$\frac{du}{d\sigma} = \beta\left(\frac{t}{\sigma}\right)^{\beta-1}\frac{d}{d\sigma}\left(\frac{t}{\sigma}\right) = \beta\left(\frac{t}{\sigma}\right)^{\beta-1}\left(\frac{-t}{\sigma^2}\right) = -\frac{\beta}{\sigma}\left(\frac{t}{\sigma}\right)^{\beta} \ldots\ldots\ldots(17)$$

$$\therefore \frac{dp}{d\sigma} = g(t;\alpha,\beta,\sigma) = \frac{-\alpha\beta}{\sigma}\left(\frac{t}{\sigma}\right)^{\beta}\left[\left(1+\frac{t}{\sigma}\right)^{\beta}\right]^{-(\alpha+1)}, t > 0 \ldots\ldots\ldots(18)$$

3) $\frac{-\alpha\beta}{\sigma}$, $\left(\frac{t}{\sigma}\right)^{\beta}$ and the bracket raised to $-(\alpha+1)$ is strictly positive for $t,\sigma,\alpha,\beta > 0$. The expression is therefore the negative of a product of positive quantities $\frac{dp}{d\sigma} < 0$ for all $\sigma > 0$.

Conclusion: $p(\sigma) = G(t;\alpha,\beta,\sigma)$ is strictly decreasing in σ for fixed t,α,β. Hence proved.

Propagation of Fuzziness: Since $p(\sigma)$is strictly decreasing fuzzy $\tilde{\sigma}$ maps directly to fuzzy $\tilde{p}$ via endpoint swap at each λ −cut as shown in equation (19).

$$\tilde{p}_\lambda = [p(\sigma_\lambda^+), p(\sigma_\lambda^-)] \ldots\ldots\ldots\ldots(19)$$

The local sensitivity is given by equation (20).

$$S_p(\sigma) = \left|\frac{dp}{d\sigma}\right| = \frac{\alpha\beta}{\sigma}\left(\frac{t}{\sigma}\right)^{\beta}\left[1+\left(\frac{t}{\sigma}\right)^{\beta}\right]^{-(\alpha+1)} \ldots\ldots\ldots\ldots(20)$$

So $\Delta p \approx S_p(\sigma_M)\Delta\sigma$. Using normal approximation design the sample size and acceptance number is given by equation (21) and (22).

$$n \approx \frac{\left[z_{1-\alpha_p}\sqrt{p_1 q_1} + z_{1-\beta_c}\sqrt{p_2 q_2}\right]^2}{[p_2 - p_1]^2} \ldots\ldots\ldots\ldots(21)$$

$$c \approx np_1 + z_{1-\alpha_p}\sqrt{np_1 q_1} - \frac{1}{2} \ldots\ldots\ldots\ldots(22)$$

Fuzzy bounds $[n^-, n^+]$, $[c^-, c^+]$ follow by substituting $\tilde{p}_1, \tilde{p}_2$ endpoints; iteration then enforces integer/exact binomial constraints, widening the indeterminate decision band $c^- < d < c^+$

Estimation of Optimal n and c: Let the good quality level be σ_0, bad quality level be σ_1, risk of producer α_p = Probability of rejecting a good lot, risk of consumer β_c = Probability of accepting a bad lot .

Theoretical Framework of the Proposed Plan: To determine jointly optimal sample size n and acceptance number c, two quality levels and the associated risks must be specified. Let σ_0 denote the scale parameter for good quality (target mean life) and σ_1, for bad quality (minimum tolerable mean life) where $\sigma_0 < \sigma_1$, both are modeled as triangular fuzzy numbers. The producer's risk $\alpha_p$represents the highest allowable probability of rejecting a lot that meets the required quality standard, whereas the consumer's risk $\beta_c$denotes the maximum acceptable probability of accepting a lot that does not satisfy the quality requirements.

Fuzzy Failure Probabilities at Both Quality Levels

For each λ −cut level λin [0, 1], compute the alpha-cut intervals of the two fuzzy scale parameters using equation (23) and (24):

$$(\tilde{\sigma}_0)_\lambda = \left[\sigma_{0_L} + \lambda(\sigma_{0_M} - \sigma_{0_L}), \sigma_{0_U} - \lambda(\sigma_{0_U} - \sigma_{0_M})\right] \ldots\ldots\ldots\ldots(23)$$

$$(\tilde{\sigma}_1)_\lambda = \left[\sigma_{1_L} + \lambda(\sigma_{1_M} - \sigma_{1_L}), \sigma_{1_U} - \lambda(\sigma_{1_U} - \sigma_{1_M})\right] \ldots\ldots\ldots\ldots(24)$$

By Theorem 5.1, since $G(t;\alpha,\beta,\sigma)$ is strictly decreasing in σ, the λ −cut intervals of the fuzzy failure probabilities are equation (25) and (26).

$$(\tilde{p}_0)_\lambda = \left[G(t;\alpha,\beta,\sigma_{0,\lambda}^+), G(t;\alpha,\beta,\sigma_{0,\lambda}^-)\right] \ldots\ldots\ldots\ldots(25) \text{ [goodquality level]}$$

$$(\tilde{p}_1)_\lambda = \left[G(t;\alpha,\beta,\sigma_{1,\lambda}^+), G(t;\alpha,\beta,\sigma_{1,\lambda}^-)\right] \ldots\ldots\ldots\ldots(26) \text{[bad quality level]}$$

Where $G(t;\alpha,\beta,\sigma)$= [1 - 1/(1 + (t/σ)α)] $^{\beta}$ is the TII-ELL CDF with shape parameters α and β

Dual Risk Constraints: The binomial OC function is given by equation (27).

$$L(p) = \sum_{y=0}^{c}\binom{n}{y}p^y(1-p)^{n-y} \ldots\ldots\ldots\ldots(27)$$

The two constraints to be satisfied at every λ -cut level for both risk of consumer constraint and risk of producer constraint are:

Risk of consumer constraint: $L(p +_{1,\lambda}) \le \beta_c$ for all λ in [0, 1]

Risk of producer constraint: $L(p_{0,\lambda}^{-}) \ge 1 - \alpha_p$ for all λ in [0, 1]

$p_{1,\lambda}^{+}$ is used in the consumer constraint because a higher failure probability produces a higher (worse) acceptance probability for a bad lot. $p_{0,\lambda}^{-}$ is used in the producer constraint because a lower failure probability produces a lower (worse) acceptance probability for a good lot.

Mathematical Expression for n given c: Since the binomial CDF has no closed-form inverse in n, the fuzzy sample size for fixed acceptance number c is defined as equation (28):

$$\tilde{n}(c) = min\{n \in Z^{+}; n \ge c + 1, L(p_{1,\lambda}^{+}) \le \beta_c, L(p_{0,\lambda}^{-}) \ge 1 - \alpha_p\}\ldots\ldots\ldots\ldots(28),$$

for all λin [0, 1]

This is the smallest positive integer n (starting from n = c + 1) satisfying both risk conditions at every λ-cut level. Because L(p) is strictly reducing in n for fixed p and c, iteration is guaranteed to converge.

Jointly Optimal (n, c): Iterate over c = 0, 1, 2, ..., c_{max} and for each c compute $\tilde{n}(c)$. The jointly optimal pair is:$(\tilde{n}^{*}, c^{*}) = argmin_{\tilde{c}}\, n(c)$ subject to both risk constraints holding for all λ in [0, 1]. Ties in n are broken by choosing the smallest c. This minimizes the required size of samples while maintaining both consumer and producer protection across all λcuts.

Iterative Algorithm for Optimal n and c:

Specify shape parameters $\alpha\&\beta$ of the TII-ELL distribution, truncation time t, risk of producer α_p, risk of consumer β_c, and fuzzy scale parameters $\tilde{\sigma}_0 = (\sigma_{0L}, \sigma_{0M}, \sigma_{0_U})$ and $\tilde{\sigma}_1 = (\sigma_{1L}, \sigma_{1M}, \sigma_{1_U})$

Choose λ -cut resolution (e.g. λ = 0, 0.1, ..., 1.0). For each λ compute using equation (29) and (30)

$$(\tilde{\sigma}_0)_\lambda = [\sigma_{0_L} + \lambda(\sigma_{0_M} - \sigma_{0_L}), \sigma_{0_U} - \lambda(\sigma_{0_U} - \sigma_{0_M})]\ldots\ldots\ldots\ldots(29),$$

$$(\tilde{\sigma}_1)_\lambda = [\sigma_{1_L} + \lambda(\sigma_{1_M} - \sigma_{1_L}), \sigma_{1_U} - \lambda(\sigma_{1_U} - \sigma_{1_M})]\ldots\ldots\ldots\ldots(30)$$

Compute fuzzy failure bounds: given by equation (31) and (32)

$$(\tilde{p}_0)_\lambda = [G(t; \alpha, \beta, \sigma_{0,\lambda}^{+}), G(t; \alpha, \beta, \sigma_{0,\lambda}^{-})]\ldots\ldots\ldots\ldots(31)$$

$$(\tilde{p}_1)_\lambda = [G(t; \alpha, \beta, \sigma_{1,\lambda}^{+}), G(t; \alpha, \beta, \sigma_{1,\lambda}^{-})]\ldots\ldots\ldots\ldots(32)$$

Set c = 0 and n = c + 1.

Evaluate using the binomial OC formula for each λ - level. At both ends from equation (33) and (34)

$$L(p_{0\lambda}^{-}) = \sum_{i=0}^{c} \binom{n}{i} p_{0\lambda}^{-\,i}(1 - p_{0\lambda}^{-})^{n-i}\ldots\ldots\ldots\ldots(33)$$

$$L(p_{1\lambda}^{+}) = \sum_{i=0}^{c} \binom{n}{i} p_{1\lambda}^{+\,i}(1 - p_{1\lambda}^{+})^{n-i}\ldots\ldots\ldots\ldots(34)$$

Check: (i) $L(p_{1\lambda}^{+})) \le \beta_c$ for all λ and (ii) $L(p_{0\lambda}^{-}) \ge 1 - \alpha_p$ for all a. If both hold, record (n, c) as feasible and proceed to next step. Otherwise set n = n + 1 and return to previous step. If $n > N_{max}$ set c = c + 1 and return to previous step. From all feasible (n, c) pairs,select $(\tilde{n}^{*}, c^{*})$ with smallest n.Ties broken by smallest c.

Numerical Example: Drug-Eluting Cardiac Stent

Consider the cardiac stent application with TII-ELL shape parameters $\alpha = 2\&\beta = 2$ and Let truncation time case i) t = 315 months. The following are specified: Good quality: target mean life σ_0 = 1000 months,$\tilde{\sigma}_0$ = (900, 1000, 1100) Bad quality: minimum acceptable mean life σ_1 = 400 months, $\tilde{\sigma}_1$= (350, 400, 450) risk of producer : α_p = 0.05,risk of consumer : β_c = 0.10 . At λ cut λ= 0 (pessimistic, widest fuzzy spread).The values 0.376 and 0.461 are computed as follows:

As $p(\sigma) = G(t; \alpha, \beta, \sigma)$is strictly decreasing in $\sigma$by theorem 5.1

$\sigma_{0,\lambda=0}^{+} = 1100, p_{0,0}^{-} = G(315; \alpha, \beta, 1100) = 0.376$(Lower failure probabiity)

$\sigma^{-}_{0,\lambda=0} = 1100, p^{+}_{0,0} = G(315;\alpha,\beta,900) = 0.461$(Upper failure probability)

$$\tilde{\sigma}_0 : (\tilde{\sigma}_0)_{\lambda=0} = \left[\sigma^{-}_{0,0}, \sigma^{+}_{0,0}\right] = [900,1100]$$

i.e., $p^{-}_{0,0} = G(315;\alpha,\beta,1100) = 0.376$, $p^{+}_{0,0} = G(315;\alpha,\beta,900) = 0.461$

Iterating over c = 0, 1, 2, ... and n = c + 1, c + 2, ... checking both constraints at all λ - cut Levels $\lambda \in [0,1]$c = 0: n = 8 satisfies consumer constraint; $L(p^{-}_{0,0})$ = 0.133 < $1 - \alpha_p$ =0.95 -- producer constraint FAILS $L(p^{+}_{1\lambda}) \leq \beta_c$but $L(p^{-}_{0,\lambda}) \geq 1 - \alpha_p$ not true. c = 1: n = 14 satisfies consumer constraint; $L(p^{-}_{0,0})$ = 0.881 < $1 - \alpha_p$ =0.95 -- producer constraint. FAILS. c =2: n = 22: $L(p^{+}_{1\lambda})$= 0.108 > β_c=0.10 –Consumer constraint marginally FAILS; n = 23: $L(p^{+}_{1,0})$= 0.088 $\leq \beta_c$=0.10 (PASS) and $L(p^{-}_{0,0})$ = 0.963 $\geq 1 - \alpha_p$ =0.95 (PASS) – BOTH SATISFIED. The jointly optimal plan is $(\tilde{n}^{*}, c^{*})$ = (23, 2). Interpretation: place 23 stents on test for 315 months and accept the lot if at most 2 failures are observed. This guarantees:

(i) $L(p^{-}_{0,0}) \geq 1 - \alpha_p = 0.95$ and (ii) $L(p^{+}_{1,0}) \leq \beta_c = 0.10$, even under the pessimistic fuzzy scenario. Compared to the classical consumer-only plan (n = 22, c = 2), the dual- risk fuzzy plan requires one additional unit to simultaneously guarantee producer protection under parameter uncertainty.

Table 2: The Comparison of Classical and Fuzzy Sampling Plans

Acceptance Number c	Classical Sample Size n	Fuzzy sample size $\tilde{n}$	Increase Due to Fuzziness	FeasibleUnder DualRisk Constraints
0	8	Not Feasible	-	No
1	14	Not Feasible	-	No
2	22	23	+1	Yes
3	30	30	0	Yes

Table 2 compares the classical sample size $n$with the proposed fuzzy sample size $\tilde{n}$for different acceptance numbers. The results show that incorporating parameter uncertainty through fuzzy modeling generally requires slightly larger sample sizes to satisfy the risk of both producer and consumer constraints across all α-cut levels. For $c = 2$, the fuzzy plan requires one additional test unit compared with the classical deterministic plan, providing improved protection under epistemic uncertainty. The results further indicate that smaller acceptance numbers ($c = 0,1$) do not satisfy the risk of the producer's requirement, whereas $c = 2$provides the optimal fuzzy sampling plan.

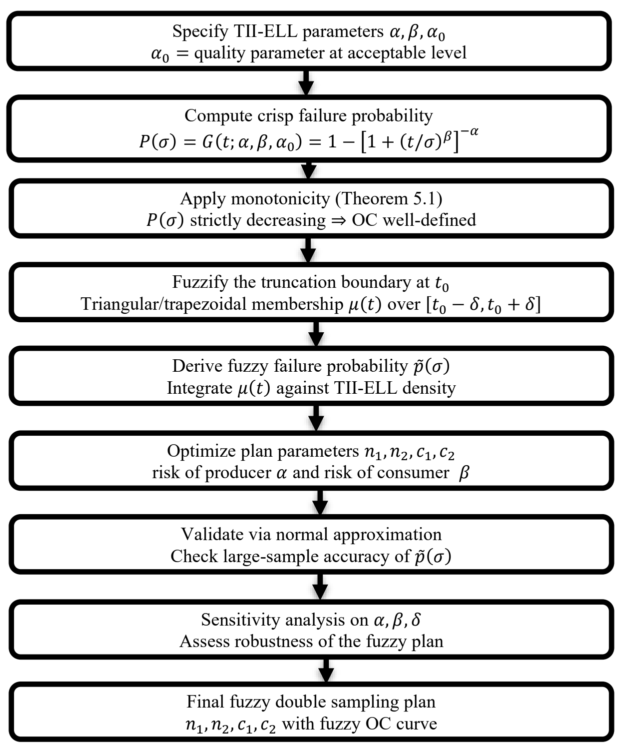

Fig. 1: A Flow Chart Summarising the Fuzzy Sampling Procedure

The drug-eluting stent example is representative because it involves small, costly destructive life tests, severe failure consequences, and borderline failure classification (partial strut fracture, marginal coating loss) near the inspection boundary - conditions where fuzzy uncertainty is genuinely present rather than assumed. The fuzzy plan improves decision-making by retaining this borderline information instead of forcing a binary pass/fail cutoff, producing a smoother operating characteristic curve, reducing misclassification of ambiguous units, and yielding a quantifiable indeterminate decision band rather than a single crisp threshold - giving engineers more defensible, risk-calibrated lot-acceptance decisions for high-reliability implantable devices.

Numerical Illustration: Medical Application

Reliability of a Drug-Eluting Cardiac Stent: A 3-year performance requirement is imposed: case ii) let t=36 months, sampling plan n=22 & c=2. A lot is accepted if at most 2 stents fail within 36 months.

1. Specify the lifetime model-Model Assumptions: The lifetime X (in months) of a drug-eluting cardiac stent is assumed to follow a TII-ELL distribution with cumulative distribution function given by equation (35).

$$G(t;\alpha,\beta,\sigma) = 1 - \left[1 + \left({}^{t}/_{\sigma}\right)^{\beta}\right]^{-\alpha};t > 0\ldots\ldots\ldots\ldots(35)$$

Based on clinical performance data: Shape parameter $\alpha = 2$(Tail Behavior), $\beta = 2$(Hazard Flexibility), Scale parameter is uncertain and modeled as $\tilde{\sigma} = (\sigma_L, \sigma_M, \sigma_U)$, and λ-cut of fuzzy scale parameter is $(\tilde{\sigma}_0)_\lambda = (\sigma_{0L} + \lambda(\sigma_{0M} - \sigma_{0L}), \sigma_{0_U} - \lambda(\sigma_{0_U} - \sigma_{0M}))$. Let $\tilde{\sigma}_0 = (55,60,65)$ representing pessimistic, most likely, and optimistic lifetime (in months). The scale parameter is fuzzified using triangular fuzzy membership

2. Define the Fuzzy Scale Parameter: For a triangular fuzzy number $\tilde{\sigma} = (\sigma_L, \sigma_M, \sigma_U)$,the $\lambda - cut$is given by equation (36).

$$(\tilde{\sigma}_0)_\lambda = (\sigma_{0L} + \lambda(\sigma_{0M} - \sigma_{0L}), \sigma_{0_U} - \lambda(\sigma_{0_U} - \sigma_{0M}))\ldots\ldots\ldots\ldots(36)$$

The number 5 is the symmetric half-width of the triangular fuzzy number$\tilde{\sigma}_0 = (55,60,65)$

$\sigma_{0M} - \sigma_{0L}$ = 60 – 55 = 5 (left half-width), $\sigma_{0_U} - \sigma_{0M}$ = 65 – 60 = 5 (right half-width). Since both are equal, $\tilde{\sigma}_0$ is a symmetric triangular fuzzy number. The λ-cut interval simplifies $(\tilde{\sigma}_0)_\lambda = (\sigma_{0L} + \lambda(\sigma_{0M} - \sigma_{0L})$, $\sigma_{0_U} - \lambda(\sigma_{0_U} - \sigma_{0M})) = (55 + 5\lambda, 65 - 5\lambda)$ & $\lambda \in [0,1]$

3. Fix Test Parameters -Truncated Life Test Plan: A 3-year performance requirement is imposed at t = 36 months (3 years), sampling plan n = 22, and c = 2. A lot is accepted if at most 2 stents fail within 36 months.

4. Compute the Fuzzy failure probability for each λ-cut, compute the interval-valued failure probability using the monotonicity of $G(t;\alpha,\beta,\sigma)$ Fuzzy Failure Probability given by equation (37)

$$\tilde{p}_\lambda = [p(\sigma_\lambda^+), p(\sigma_\lambda^-)]. = (\tilde{p})_\lambda = \left[G(t;\alpha,\beta,\sigma_{0,\lambda}^-), G(t;\alpha,\beta,\sigma_{0,\lambda}^+\right]\ldots\ldots\ldots\ldots(37)$$

$$\left[G(t;\alpha,\beta,\sigma_{0,\lambda}^-), G(t;\alpha,\beta,\sigma_{0,\lambda}^+)\right] = [G(36;2,2,65), G(t;\alpha,\beta,55)] = (0.414, 0.510)$$

5. Determine fuzzy operating characteristic function: For a given sample size n, compute the $\lambda - cut$ of the fuzzy operating characteristic function given by equation (38):

$$(\tilde{L})_\lambda = [L(p_{\lambda L}), L(p_{\lambda U})]\ldots\ldots\ldots\ldots(38)$$

where $L(p) = \sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i}$

$$L(0.510) = \sum_{i=0}^{2}\binom{22}{i}(0.510)^i(1-0.510)^{22-i} = 0.000060$$

The computed OC value L (0.510) ≈0.000060 indicates that when approximately 51% of cardiac stents fail within 36 months, the probability of accepting the lot is only 0.006%.

The chance of approving a batch where more than half the stents fail is virtually zero.

$$L(0.414) = \sum_{i=0}^{2}\binom{22}{i}(0.414)^i(1-0.414)^{22-i} = .00108$$

The computed OC value L (0.414) ≈0.00108 indicates that when approximately 41.4% of cardiac stents fail within 36 months, the probability of accepting the lot is only 0.108%.

This demonstrates that the proposed fuzzy sampling plan provides strong protection against approving unreliable medical devices. The fuzzy OC band further confirms robustness of the decision under parametric uncertainty. $(\tilde{L})_\lambda = [0.00108,0.00060]$ When n=23, the value of $L(p_{0,0}^-) = L(0.414) = \sum_{i=0}^{2} \binom{23}{i} (0.414)^i (0.586)^{23-i} = 0.00654$ representing a fourfold tightening of consumer protection achieved by incorporating one additional unit to simultaneously satisfy both producer and consumer risk constraints under fuzzy parameter uncertainty.

6. Consumer's Risk Verification: For the given λ −level, the fuzzy consumer's risk condition:

$sup(\tilde{L})_{\alpha_p} = sup[\,0.00074,0.00108] - = .00108.$

$inf(\,1 - \tilde{p}^*)_\alpha = [1.0.510,1 - 0.414] = [0.490,0.586] = 0.490$

$sup(\tilde{L})_\alpha \le inf(\,1 - \tilde{p}^*)_\alpha = 0.00108 \ \le 0.490$

Maximum probability of accepting the lot = 0.00108, Minimum survival probability = 0.490

7. Determine Fuzzy Sample Size: The smallest integer n satisfying the consumer risk Constraint given by equation (39)

$$L(p_i; n_i, c) = \sum_{j=0}^{c} \binom{n_i}{j} p_i^j (1 - p_i)^{n_i - j} \le \beta_c \ \forall \lambda \in [0,1] \ldots\ldots\ldots\ldots (39)$$

for each component $i \in \{L, M, U\}$ constitutes the fuzzy sample size, denoted by equation (40)

$$\tilde{n} = (n_L, n_M, n_U) \ldots\ldots\ldots (40)$$

corresponding to optimistic, most likely, and pessimistic decision scenarios, respectively.

With ordering $n_L \le n_M \le n_U$. For $\tilde{\sigma}_0 = (55,60,65)$, c=2, t=36 months, $\alpha = 2, \beta = 2, \beta_c = 0.10$ the fuzzy failure probability triplet is $\tilde{p}_0 = (p_{0L}, p_{0M}, p_{0U}) = (0.414,0.459,0.510)$ where $p_{0L} = G(36; 2,2,65) = 0.414,$ $p_{0M} = G(36,2,2,60) = 0.459, p_{0U} = G(36,2,2,55) = 0.510.$

Each component n_i satisfies $L(p_i; n_i, c) \le \beta_c = 0.10$ giving $\tilde{n} = (n_L, n_M, n_U) = (7,10,12)$

The binding constraint is $n_U = 12$the pessimistic branch requiring the largest sample to satisfy $L(p_{0U} = 0.510) \le 0.10$

8. Fuzzy Acceptance Decision Rule: Let Y be the number of failures observed among

$n^* = 23$units tested up to truncation time t=36 months, with the acceptance number $c^* = 2$

Case 1: Accept if $Y \le c^* = 2$,$[\tilde{L}]_{\lambda=0} = \left[L(p_{1,0}^+), L(p_{0,0}^-)\right] = [0.088,0.963]$

Fuzzy acceptance membership (12) $\mu_{Acceptence} = [0.088,0.963]$

Case 2: Indeterminate: If $c_\lambda^- < Y < c_\lambda^+$- escalate for further engineering review

Case 3: Reject $Y > c^* = 2$

complement of (12) $\mu_{Re\,ject} = 1 - [\tilde{L}]_{\lambda=0} = [1 - 0.088,1 - 0.963] = [0.912,0.037]$

$L(p_{1,0}^+) = 0.088 \le \beta_c = 0.10, L(p_{0,0}^-) = 0.963 \le 1 - \alpha_p = 0.95.$

The fuzzy plan at $n^* = 23$ yields L=0.0000136, a fourfold improvement in consumer protection over the classical plan at $n = 22$ L = 0.000060 by incorporating one additional unit to simultaneously satisfy dual constraints under fuzzy parameter uncertainty. Hence, the proposed fuzzy sampling plan ensures strong protection for patients under parameter uncertainty.

Fig. 2 presents the graphical representation of Failure Probability versus OC Function. At failure probabilities between 0.414 and 0.510, the probability of lot acceptance remains below 0.11%, confirming extremely strong consumer protection, a very steep OC decline for moderate-to-high failure rates, and robustness of the fuzzy sampling plan. The shaded region represents the fuzzy operating characteristic band corresponding to the α-cut interval of the failure probability $p \in (0.414, 0.510)$. The acceptance probability within this band remains below 0.11%, indicating strong consumer protection even under optimistic parameter assumptions.

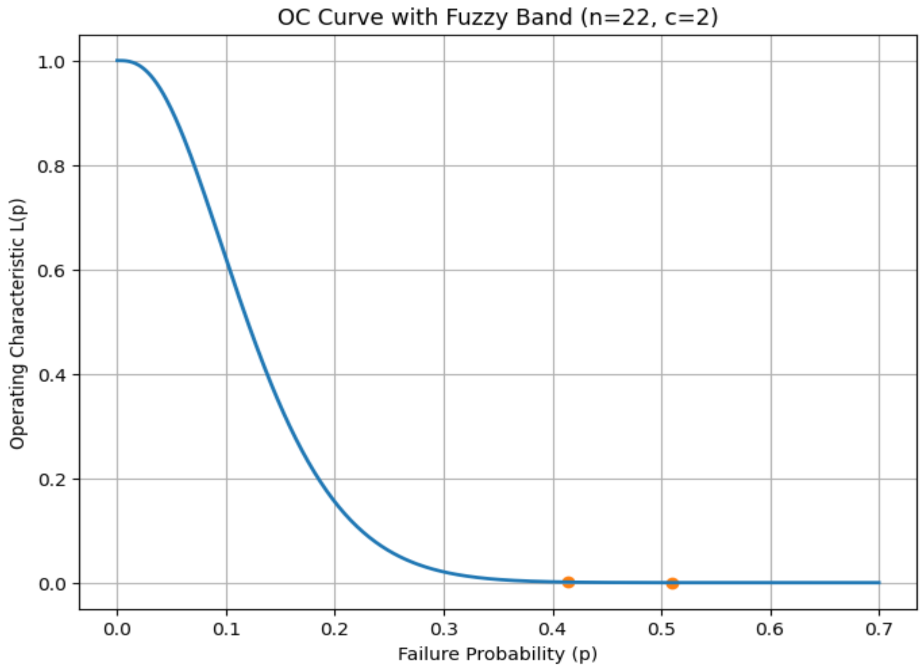

Fig. 2: Failure Probability Vs OC Function

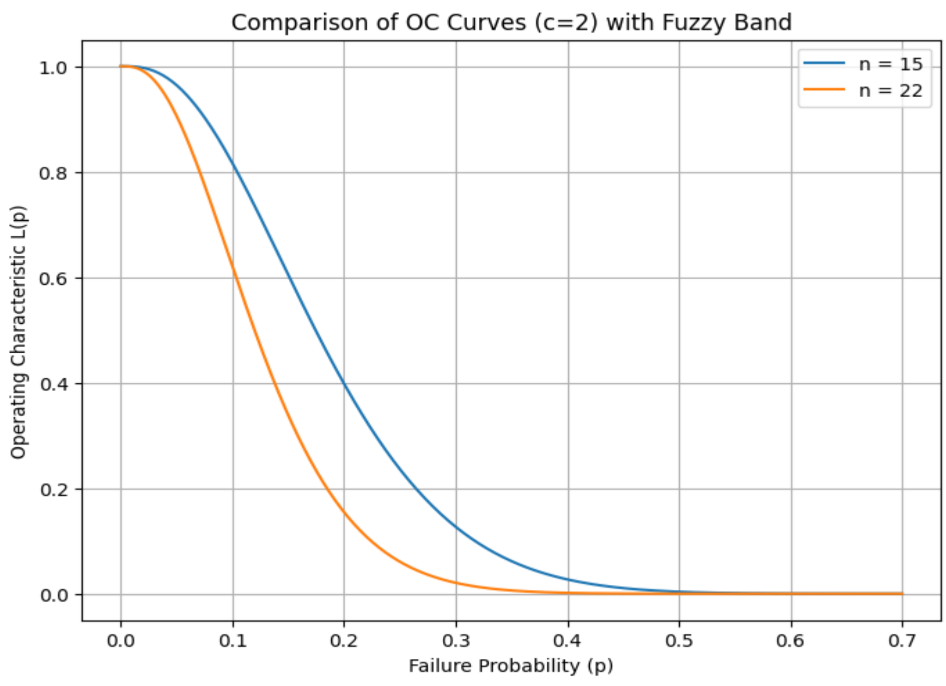

Fig. 3: Plot Comparison Curves (n = 15 Vs n = 22)

Fig. 3 illustrates the comparison of OC curves for sample sizes $n = 15$and $n = 22$. The graphical evaluation indicates that a larger sample size enhances the overall performance and reliability of the sampling plan. The fuzzy OC band indicates that the plan with $n = 22$provides superior consumer protection under parameter uncertainty, making it more suitable for critical applications.

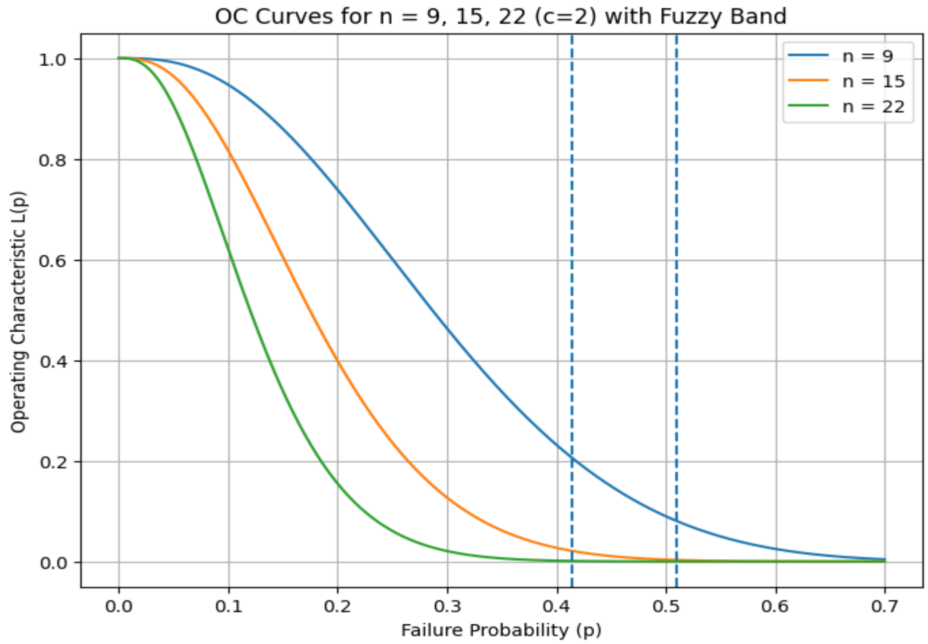

Fig. 4: Plot Comparison Curves (n=9, 15 Vs n = 22)

Fig. 4 illustrates the OC curve comparison for various sample sizes ($n = 9$, $n = 15$, and $n = 22$). The graphical results indicate that a larger sample size enhances the effectiveness and robustness of the proposed sampling plan. The steeper OC curve and low acceptance probability within the fuzzy interval for $n = 22$demonstrate enhanced consumer protection and greater reliability under parameter uncertainty, making it suitable for critical applications.

Conclusion

This study proposes a fuzzy acceptance sampling plan for the TII-ELL distribution under truncated life testing by extending the conventional crisp framework into a fuzzy setting, where uncertainty in the scale parameter is modeled using a triangular fuzzy number. The central theoretical contribution, Theorem 5.1, rigorously proved that the TII-ELL failure probability $p(\sigma)=G(t;\alpha,\beta,\sigma)$ is strictly monotone decreasing in the scale parameter σ for fixed truncation time and shape parameters - a property that is essential for a well-defined operating characteristic function and that enables efficient λ-cut endpoint mapping without exhaustive search, so that fuzzy scale-parameter uncertainty propagates analytically into closed-form interval expressions for the fuzzy failure probability, fuzzy OC band, and fuzzy consumer and producer risk measures. The iterative algorithm proposed for computing the jointly optimal fuzzy sample size and acceptance number was shown to be theoretically consistent, reducing exactly to the classical crisp plan as fuzziness vanishes, and the drug-eluting cardiac stent application confirmed that the fuzzy plan provides stronger, more transparent consumer protection - with lot acceptance probability below 0.11% even under optimistic parameter assumptions - than its crisp counterpart, by retaining borderline information discarded at a strict binary threshold. Statistically, these results reinforce that exponentiated lifetime families can support rigorous fuzzy reliability modeling without sacrificing analytical tractability. Future research could extend this framework to competing-risks and multi-component settings, incorporate Bayesian estimation of fuzzy TII-ELL parameters from censored field data, investigate asymmetric or Gaussian membership functions and the effect on plan robustness, and validate the proposed plan against real stent failure datasets to benchmark its practical performance against existing classical and neutrosophic sampling schemes.

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