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Different methods of image mapping, its advantages and disadvantages

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Abstract

Image mapping is a growing branch of image processing science [1]. Geometric transformation of digital images is the main discussed subject in this branch. Geometric transformation of points located in a source image pass to different coordinates in destination image. Mapping can be referring to a simple geometric transformation such as moving or complex conversions that even they cannot be expressed in a closed form. In this study, different mapping methods, their advantages and disadvantages are discussed.

Keywords: image mapping, backward method, affine, perspective

Introduction

Historically mapping was conducted the first time in 1960 on an analogue image by using optical systems. In 1987, significant progresses were achieved in this field. The optical systems with this unique advantage, work with the speed of light. The limitation of these systems is in their lower control and flexibility. Digital computers do not have these limitations and can do the mapping with very good precision and quality. Geometric conversions of digital images were used for a first time in the field of remote Sensing. From mapping applications can be noted to the applications of remote sensing, radiology, computer graphics, morphing, rendering, tracking [2, 3], image stabilization [4,1, 5], improving the image [6] and compression the remainder of video image using the interpolation between frames [10].

1. Mapping procedures

Image mapping can be done in two ways, Forward and Backward, that each has advantages and disadvantages.

1.1 Forward Method

In the Forward method, transfer function mapping is applied to the source image and each pixel of the resource image is mapping to the point of destination image. Figure1.2 shows this method.

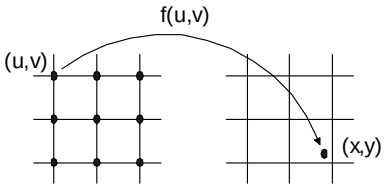


Figure 1: Forward Mapping

In Figure 1 (u, v) coordinates of a pixel indicates in the source image. This pixel by mapping function f (u, v) to the point with coordinates (x, y) is mapped in the destination image. Since the image is digital and pixel coordinates are integers, often at the mapped point, there is no pixel .In this case, the mapped value is attributed to one or more pixels that have closest distance. If the mapping value is attributed to a few pixels, first it should be specified their numbers or maximum distance from a mapping site, and secondly, a coefficient that is given to each pixel should be determined depending on the distance

In other words, for each pixel of destination image, should be defined a place range or field that if a value was mapped to a point within this range was. This pixel will be affected by it and it will also be defined on the basis of these influences rates. First, it may be not mapped no values in the field of effects of destination pixels, which will be caused dark spots in the image. Second, the number of mapped points in the influence range of each pixel is uncertain and this issue causes a difficulty for weightings. In fact, for each of the image destination pixels should be considered a buffer that are maintained all the mapped points in the range of its influence with each intervals and finally determine the true value of the pixel, or the destination pixel is read of a memory and after modifying again is placed into its location. Implementing such a procedure requires high processing costs that cause to prolong the processing time and also to increase the hardware volume. However, in applications where high precision is not required, this method because of the mentioned advantages, is very suitable for implementation

The advantage of this method is that it does not need to supply the total input image in order to start the process, but in general this method is not suitable for implementation and in action is facing with two flaws.

1.2 Reversed method

In reversed method, to calculate the value of a destination image pixel, reversed transfer function is mapped to the coordinates of that pixel and the coordinates of the corresponding point in the source image is calculated. Figure 2 shows the reversed method. Pixel of the destination image with its coordinates (x, y) is mapped to the backward function f to the point with coordinates (u, v) in the source image. Such as Forward method there is usually no pixel at this point. In this case, or the value of the nearest pixel to the mapping point as the destination image pixel value is considered or with the help of interpolation of several points of its surrounding the pixel values are calculated.

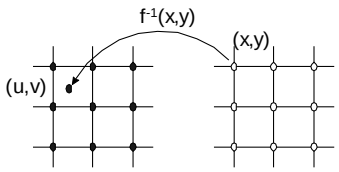


Figure 2: Backward Mapping

Defects listed for the Forward method will never happen here. This method is suitable for implementation. Most reported implementations in essays have been used of this method [6, 10, 9, 8, 11, and 7].

2. Mapping function

Mapping function is a relation that is connected the coordinate of points located in an image and mapped image. According to the application of the mapping is determined the kind of function. In this chapter focus is on conversions that are the most applicable. These conversions have usually closed mathematical forms, and most important of them, are the affine transformations, perspective, bilinear and polynomial, which their explanations will come. Of course there are conversions that cannot be considered a closed form for them that will be also investigated the manner of implementing such conversions.

2.1 Affine transformations

Affine public display is as follows:

$$[x, y, 1] = [u, v, 1]\begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \quad (1)$$

In other words, this conversion may be showed as relationship (2).

$$\begin{cases} x = a_{11}u + a_{21}v + a_{31} \\ y = a_{12}u + a_{22}v + a_{32} \end{cases} \quad (2)$$

Affine transformation is equivalent to the Parallel Plane Projection of source image on the destination image. For this reason, parallel lines remain parallel after mapping. Also the points with same distance are preserved after mapping (although the actual distance between the two images may different)

As this conversion has six parameters are defined by three points. For this reason, a triangle is mapped to any arbitrary triangle, but cannot be mapped a quadrangle to any arbitrary quadrilateral. In this conversion, the straight lines are preserved and sometimes are used of this feature in the implementation of this transformation [3]. Simpler transformations are derived from this conversion that will be expressed the public form for each in the following. Transmission is a conversion that u and v with constant values are added and the conversion transition matrix is as follows:

$$[x, y, 1] = [u, v, 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_u & T_v & 1 \end{bmatrix} \quad (3)$$

Rotation, all points are turned around the origin as an amount of the angle θ

$$[x, y, 1] = [u, v, 1] \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

4. Resize makes the image larger or smaller

$$[x, y, 1] = [u, v, 1] \begin{bmatrix} S_u & 0 & 0 \\ 0 & S_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

When x has linear dependence to u and y takes return same amount of v same or vice versa, convert that is happen cutting and can be expressed in the following form.

$$[x, y, 1] = [u, v, 1] \begin{bmatrix} 1 & H_u & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [x, y, 1] = [u, v, 1] \begin{bmatrix} 1 & 0 & 0 \\ H_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6) \text{ and } (7)$$

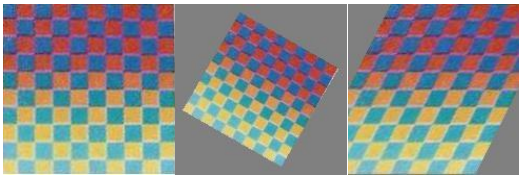
Three transformations, rotation and resizing and conversion can be combined and can be achieved a conversion that will be included all of them.

As mentioned in above, to identify the affine conversion parameters, three clear points in the source image and their locations after mapping are sufficient. If three points are clear (x_k,y_k) and (u_k,v_k) for k = 0, 1, 2 then we have:

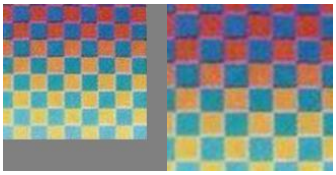
$$\begin{bmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} = \begin{bmatrix} u_0 & v_0 & 1 \\ u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \quad (9)$$

This relationship can be expressed as X = UA.

Finally, a few examples of affine transformations have been shown in Figure 3.



(A) (B) (C)



(E) (D)

Figure 3 shows a few examples of affine transformations: A) Original image B) Rotation of the -30 degrees and Converting the scale of 0.66 C) 30 degrees horizontal cutting D) Horizontal and vertical transmission E) Changes in 1.5 scales (from [1])

2.2 Converting the perspective

Public display of perspective transformation is as follows:

$$[x', y', w'] = [u, v, w]\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (10)$$

$$x = x'/w' \quad , \quad y = y'/w'$$

Perspective conversion or projection mapping, occurs when $[a_{13},a_{23}]^T$ to be not zero. In this conversion, straight lines after mapping are preserved. Only parallel lines remain that are parallel to the radiation plate. The group of other parallel lines converges towards a Vanishing Point. Using Equation 10 Forward Mapping function of this conversion will be as follows:

$$\begin{cases} x = \dfrac{x'}{w'} = \dfrac{a_{11}u + a_{21}v + a_{31}}{a_{13}u + a_{23}v + a_{33}} \\ y = \dfrac{y'}{w'} = \dfrac{a_{12}u + a_{22}v + a_{32}}{a_{13}u + a_{23}v + a_{33}} \end{cases} \qquad (11)$$

This conversion does not have the parameter that by normalizing the general matrix T1 according to a_{33} decreases to eight parameters. Therefore, this conversion will be determined with four-points and is capable to map a tetrahedron to any arbitrary quadrilateral. Figure 4 shows two examples of perspective conversion.

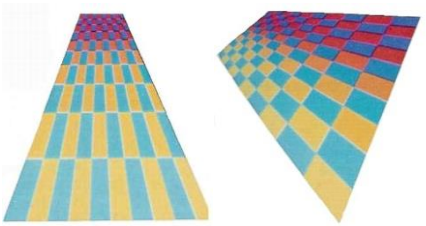


Figure 4: examples of perspective mapping

As affine transformations, for this transformation also can be calculated backward transform. Reverse of perspective transformation will be as follows.

$$[u,v,w]=[x',y',w']\begin{bmatrix}a_{22}a_{23}-a_{23}a_{32}\\a_{23}a_{31}-a_{21}a_{33}\\a_{21}a_{32}-a_{22}a_{31}\end{bmatrix} \quad (12)$$

To obtain the coefficients, after normalizing the public matrix T1 in terms of a33, Equation 11 is rewritten as follows. If (x_k,y_k) and (u_k,v_k) for k = 0,1,2,3 are four distinct points of source and destination, can be formed the equation 14 by using the relationship 13.

$$\begin{cases} x = a_{11}u + a_{21}v + a_{31} - a_{13}ux - a_{23}vx \\ y = a_{12}u + a_{22}v + a_{32} - a_{13}uy - a_{23}vy \end{cases} \quad (13)$$

In this equation, $A=[a_{11}\ a_{21}\ a_{31}\ a_{12}\ a_{22}\ a_{32}\ a_{13}\ a_{23}]^T$ and $X=[x_0\ x_1\ x_2\ x_3\ y_0\ y_1\ y_2\ y_3]^T$ are two vectors in forms and.

$$\begin{bmatrix} u_0 & v_0 & 1 & 0 & 0 & 0 & u_0x_0 & v_0x_0 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & u_1x_1 & v_1x_1 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & u_2x_2 & v_2x_2 \\ u_3 & v_3 & 1 & 0 & 0 & 0 & u_3x_3 & v_3x_3 \\ 0 & 0 & 0 & u_4 & v_4 & 1 & u_4y_4 & v_4y_4 \\ 0 & 0 & 0 & u_5 & v_5 & 1 & u_5y_5 & v_5y_5 \\ 0 & 0 & 0 & u_6 & v_6 & 1 & u_6y_6 & v_6y_6 \\ 0 & 0 & 0 & u_7 & v_7 & 1 & u_7y_7 & v_7y_7 \end{bmatrix} A = X \quad (14)$$

By solving this system of linear equations can be obtained coefficients of perspective mapping

2.3 transform bilinear

Public form of bilinear transformation is relationship15. The transformation has the most applications in mapping of a rectangle to an uneven quadrilateral.

$$[x, y] = [uv, u, v, 1] \begin{bmatrix} a_3 & b_3 \\ a_2 & b_2 \\ a_1 & b_1 \\ a_0 & b_0 \end{bmatrix} \quad (15)$$

In remote sensing and medical illustrator to calibrate the sensor is mapped clear points to locations that are already specified to be compensated sensor disturbances and registration is done very good. In computer graphics for use histograms, this transformation plays a major role. This transformation of straight lines vertically and horizontally, so stay right after mapping. Equidistant points on the straight lines vertically and horizontally, after mapping also remain equidistant. Lines that are not in line with the vertical arrangement are turned into second-rate curves. Figure 5 shows two examples of this transformation.

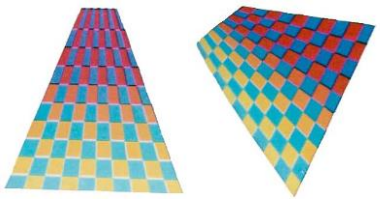


Figure 5: examples of two linear mapping

To achieve the coefficients of this conversion, as perspective conversion, four points in the source image and corresponding with them must be known in the destination image. If (u_k,v_k) and (x_k,y_k) for k =

0,1,2,3 are these four points, solving linear equations 16 will be led to calculate coefficients a_0 to a_3. b_0 to b_3 will be calculated with the same relation.

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & u_0 & v_0 & u_0 v_0 \\ 1 & u_1 & v_1 & u_1 v_1 \\ 1 & u_2 & v_2 & u_2 v_2 \\ 1 & u_3 & v_3 & u_3 v_3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad (16)$$

4.2 polynomial mapping

Geometric correction of an image requires to location conversion to reverse an Unknown distortion function. This is done using a polynomial transformation.

This function is expressed by the following general relationship.

$$\begin{cases} u = \sum_{i=0}^{N} \sum_{j=0}^{N-i} a_{ij} x^i y^j \\ v = \sum_{i=0}^{N} \sum_{j=0}^{N-i} b_{ij} x^i y^j \end{cases} \quad (17)$$

N degree is determined with regard to the application. In most applications 2 degree is sufficient, but 3 and 4 degrees are also used. 1degree of this conversion is equivalent to affine conversion. Affine conversion can be known as a combination from several specific geometric conversions such as rotation and transformation parameters can be obtained by this way. The reason has many applications in computer graphics. Unlike computer graphics in the fields of remote sensing, medical imaging and machine vision, often things are not like this.

In these fields, there is no specific model to be applied to the image, but in the input and output images, some corresponding points are clear that are said them control points or Tie points. In such conditions, conversion parameters should be achieved in such a way that conversion function is established the corresponding of control points. When the model parameters were obtained, this model is applied to the whole image and the output image is obtained in modified form. To obtain the coefficients of this conversion, coordinates of clear points are put in 17 relations until a system of equations, which usually the number of its equations is greater than the number of unknowns, is taken shape. In a polynomial N degree of the number of parameters is obtained from the 18 relationship.

$$K = \sum_{i=0}^{N} \sum_{j=0}^{N-i} 1 = \frac{(N+1)(N+2)}{2} \quad (18)$$

2.5 Public mapping

In some circumstances it may be mapped in such a way that the mapping function cannot be expressed in closed-form. In such circumstances of the LUT is used for mapping implementation. In reversed mode for

all parts of the destination images, corresponding coordinates in the source image are calculated and are placed in a memory. Memory size should be larger than the total number of image pixels. Now for mapping an image instead of pixels of m destination image, the point of source image that it's coordinate is in the m address of memory is selected.

In Forward mode, this method does not change, only the location of two images is changed and mapping function for source image is calculated. The main advantage of this method is that it can perform mapping functions with any complexity. If the mapping function is the same for all images, this method eliminates the need to the time-consuming and high-volume calculations and the mapping with high-speed can be performed with this method [2, 11].

Conclusion

Bilinear conversion in remote sensing and medical illustrator to calibrate the sensor, clear points are mapped to locations that are already specified to be compensated sensor disturbances and registration is done very good. In computer graphics for use histograms, this conversion plays a key role. Public mapping can perform mapping functions of any complexity. If the mapping function is the same for all images, this method eliminates the need to the time-consuming and high-volume calculations and the mapping with high-speed can be performed with this method and in contrast to affine conversion the perspective conversion can be mapped a tetrahedron to any arbitrary quadrilateral. Forward methods has two major disadvantage, the first is this in the sphere of influence of some destination pixels are not mapped to any value, which will be caused dark spots in the image. Second, the number of mapped points in the sphere of influence of each unclear pixel and it makes problem for weighting. Listed disadvantages for the Forward method will never happen in reverse method. And due to the reduction of implementation time is very suitable.



Figure 6: simulated mapping function (perspective, polynomials)

Generally for conversions such as transport, scale changes, rotation, a second-degree polynomial is sufficient [11]. For more local control, affine conversion and piece polynomial that their parameters are in different parts has many applications.

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