International Academic Institute for Science and Technology

International Academic Journal of Science and Engineering
Vol. 6, No. 1, 2019, pp. 26-38.

ISSN 2454-3896

International Academic Journal of Science and Engineering

www.iaiest.com

Investigating the effect of restraints' configuration on resistance of double layer braced barrel vaults to Progressive collapse

Seyedamin Mousavi[a], Rahmatelah Najafpour[a], Mohammadreza Sheidaii[b]

[a] *Master Student of Civil Engineering, Urmia University, Urmia, Iran*

[b] *Professor of Civil Engineering, Urmia University, Urmia, Iran*

Abstract

Progressive collapse is a phenomenon in which a small localized damage spreads in different parts of the structure, and may damage the entire structure or a large part of it. Since the members in space structures only resist axial forces, these structures are more vulnerable to progressive collapse. One of the important factors in behaviour of these structures is the distribution of restraints. In this study, the effect of the configuration of supports on the resistance of double layer braced barrel vaults to progressive failure has been evaluated. To this end, five double layer braced barrel vault with height of 6.20 (m), covering area of 30 × 30 (m^2), and different support arrangement was designed. Then, critical members were identified for each load combination. By removing critical members, dynamic load was applied to the structure and its behaviour was evaluated using nonlinear time histories analysis. Both nonlinear behaviour of materials and nonlinear geometric behaviour of members are considered in numerical simulation of models. Final responses show that the failure of members with higher stress ratio, results more dynamic force on structure which may cause progressive failure. However, eliminating members with lower stress ratio don't cause progressive deterioration in structure. Furthermore, density of restraints has a great effect on the behaviour of double layer braced barrel vault and finding alternative stable path after removal of critical member.

Keywords: Progressive collapse, Space frame, Double Layer Braced Barrel Vault, Nonlinear dynamic analysis

DOI:10.9756/IAJSE/V6I1/1910003
Received: 08 March 2019/Revised: 04 April 2019/Accepted: 09 May 2019/Published: 28 June 2019

Introduction

Truss structures are structural systems consist of straight members, which can only carry tension and compression. These types of structures are suitable for variety of purposes and have been investigated by many researchers. For instance, (Arabi, Shafei, and Phares 2018) investigated the fatigue performance of sign support truss structures during their transformation to the site of construction. Additionally, they conducted a comprehensive long-term fatigue analysis on an in-service sign support structures under various loads such as wind and temperature, (Arabi 2017) and (Arabi, Shafei, and Phares 2019). They confirmed the superior fatigue performance of steel structures over aluminium structures, (Arabi, Shafei, and Phares 2017). Space truss structures are also considered as a reliable solution in space structure domain. Barrel vault is one of the oldest architectural forms of space frames used since antiquity, (Makowski 1990). The iron barrel vault, which was constructed in 1851over the central nave of the Crystal Palace for the Great Exhibition of Hyde Park in London, is considered as the earliest barrel vault constructed with modern structural material,(Makowski 1990). As the basic geometry, barrel vault is formed of single or double-curved grid surfaces. They are constructed with different range of materials including traditional ones such as steel, aluminium or timber to newer ones such as fiber-matrix composite material,(Makowski 1990). Braced barrel vaults were initially constructed as single-layer structures. However, they could be constructed in configurations with double or even triple layers of members. The use of single-layer structures is limited by stability problems especially under the asymmetric load condition,(Subramanian 1999). This can be avoided by utilizing double-layer braced barrel vaults. For larger spans, a double-layer system should be employed. Nowadays, double-layer systems are used for clear spans near to 100 m,(Makowski 1990).

The Double layer barrel vault can be designed and constructed in various configurations such as square on square, square on diagonal and diagonal on square,(Chilton 2000). In these systems, the upper and lower chord layers are braced by diagonal members. Previous studies showed that under the constant support and load condition the diagonal on square configuration is not as efficient as the other two configurations and distribution of stress does not occur evenly throughout the structure,(Kramer and Gerhardt 1988). Generally space frame systems are statically highly redundant and it is believed that after the failure of one member or part of the structure, rest of the members can easily absorb redistributed forces and easily tolerate excess loads. But, some real evidence cast doubt on the authenticity of the claim. Hartford stadium, collapsed in the United States, is an instance of this case. In this stadium, double layer space frame was used to cover an area of approximately 82×83 (m^2). The structure was destroyed on 10,000 seats on the morning of January 18, 1978, while the stadium was empty and luckily no one was hurt,(Martin and Delatte 2001).

If the load-carrying capacity of one or a set of members suddenly decreases due to buckling or other factors causing local failure in the space frame, load redistribution may be so rapid and sudden that other members will not be able to sustain the load quickly. Therefore, immediately after happening local breakdown, a temporary instability occurs in the balance path of the structure. In the search for a stable equilibrium state, the structure is suddenly transferred to a new one at the same energy level. Moving to a new state of balance involves large and sudden changes in the configuration of the structure and it is associated with intense dynamic effects exerting additional loads on the structure and thus increasing the risk of failure of whole structure.

Generally, progressive collapse can be defined as a kind of failure in which breakdown of a relatively small part of the structure, cause sequential failure or damage in whole system. Progressive deterioration is a chain propagation of failure in which, due to certain causes, the local breakdown of a relatively small area of the structure can extend to other parts of it and ultimately results in a general failure of the structure. This type of failure can be continuous deterioration of seals like Domino and that is why sometimes it is referred to the domino failure mechanisms. Since structures are usually designed to sustain common loads such as dead loads, live loads, and earthquake loads, the initial imperfection and design errors are not usually considered. Unlike the conventional design loads, initial imperfection can cause local failure in the structure and cause only one or more serious structural damage. However, a single local breakdown can spread to the entire structure and cause its full failure. In such cases, for the proper evaluation of structural failure, static analysis is not sufficient alone and dynamical effects resulting from load redistribution should be considered properly in the analysis.

Double layer braced barrel vaults are investigated in different aspects in recent years. Most of the studies are about optimal design of this structural system. (Kave and Eftekhar 2012) provided an optimal design of barrel vaults using the Improved Big Bang-Big Crunch (IBB-BC) method. Later, he utilized the charged search system (CSS) optimization algorithm to provide an optimized design of double layer barrel vaults,(Kaveh, Farahani, and Shojaei 2012). In another effort, he improved the magnetic charged system and applied it for these systems,(Kaveh, Mirzaeib, and Jafarvandb 2013). He used Colliding Bodies Optimization (CBO) and its enhanced version (ECBO) and also Lion Pride optimization algorithm for the optimization of double-layer barrel vaults,(Kaveh and Moradveisi 2016) and (Kaveh and Mahjoubi 2018). (Shin et al. 2012) studied seismic behaviour of double-layer barrel vault systems numerically and experimentally with different open angles. They found that models with an open angle of 90 degrees have the most beneficial shape of the barrel vault structures. (Yadollahi 2011) investigated the effect of initial geometric imperfection on the load bearing capacity of double layer barrel vaults. They concluded that imperfections in the critical area do not have a remarkable effect on the structure initial stiffness but decrease the maximum load bearing capacity. Moreover, (Tahamouli Roudsari et al. 2017) assessed the simultaneous effects of random distribution of initial curvature and length imperfections on the collapse behaviour and reliability of double layer barrel vaults. According to their results, these structures are highly sensitive to simultaneous random initial geometrical imperfections and their capacity decrease considerably.

Positions of the supports and their distance have significant effect on the economical design of the structures. In barrel vaults this aspect can be more important since the number of the elements and connections are high. In other hand, configuration of the supports plays an outstanding role in the resistance of these kinds of structures to progressive collapse. Although the above mentioned studies cover different aspect of structural behaviour of barrel vaults, there is limited investigation about resistance of these types of structures to progressive collapse phenomenon.

Numerical Modeling

There are many geometrical factors which can affect the performance of double layer barrel vaults. In this study the resistance of these structures is evaluated by considering various support configuration and nonsymmetrical loading condition. For that purpose, six numerical models are considered. The geometry of models includes two parallel cylindrical grid layers which the basic module size of the grid is 2×2 (m)

square panels. The structures have raise of 6.2 (m) and distance between the two grid layers is 2 (m), refer to figure 1. Radius of outer layer is 15 (m) and whole structure covers 30×30 (m^2) area. Each model contains 481 nodes and 1800 elements. The position of supports varies in each model and figure (2) shows the arrangement of supports for each model. Columns in these figures are provided for graphical purpose and they are not included in numerical models. The first model (BV1) has a dense restraint (the distance between supports are 2 meters) in two sides parallel with the longitudinal axis. The second model (BV2) is similar to the first one but is has supports with three panel intervals, (the distance between supports are 6 meters). The third sample (BV3) is similar to the two previous models but has supports at each 10 meters (five panel intervals) and the density of restraint is less than the previous models. Other three models have restraints both in longitudinal and transverse direction. Spacing of the supports in model (BV4) is 2 (m), in model (BV5) is 6 (m) and in model (BV6) is 10 (m) in all four edges of structure.

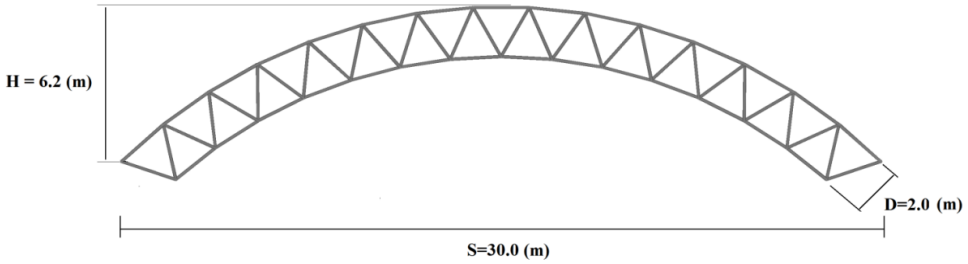
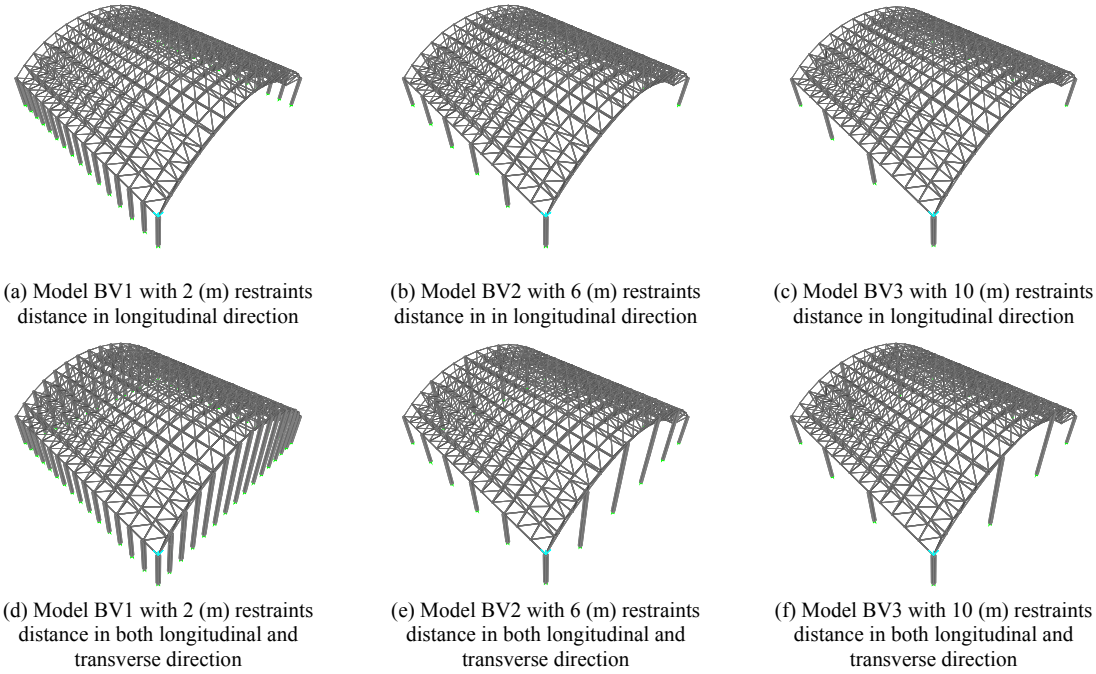


Figure .1 Front view of numerical models' geometry



(a) Model BV1 with 2 (m) restraints distance in longitudinal direction

(b) Model BV2 with 6 (m) restraints distance in in longitudinal direction

(c) Model BV3 with 10 (m) restraints distance in longitudinal direction

(d) Model BV1 with 2 (m) restraints distance in both longitudinal and transverse direction

(e) Model BV2 with 6 (m) restraints distance in both longitudinal and transverse direction

(f) Model BV3 with 10 (m) restraints distance in both longitudinal and transverse direction

Figure.2 Position of supports in numerical models

In order to design the models, 50 kg/m2 dead load for roof system and 150 kg/m2 snow load as a live load in both symmetric and asymmetric condition is considered. The load combinations are considered according to the ASCE07-16 instruction, (American society of civil engineers 2014). The AISC 360-16 code is used for designing models,(ANST/AISC 2016). The basic geometry of models established in Formian software and then transferred to SAP2000 software (CSI 2015) to be analyzed and design with various restraint configurations,(Nooshin and Disney 2002). The standard circular structural hollow sections are considered for members,(ANST/AISC 2016).Tables (1) and (2) provide property and quantity of each sections used for each layer. In these tables P refers to pipe section and the first number gives the outer diameter and second number shows the thickness of the section in millimetre. Tables also show the weight of each layer and total weight of the each model. Figure (3) illustrates the variation of the weight of each layer and also for whole structure.

Table1 Structural properties of members BV1, BV2 and BV3

Model	Top Layer			Web Layer			Bottom Layer			Total Weight
	Section	Quantity	Layer Weight	Section	Quantity	Layer Weight	Section	Quantity	Layer Weight	
BV1	P33.7×2.6	240	376.67	P42.4×2.6	900	579.19	P33.7×2.6	210	227.2	1183.06
	P60.3×2.9	28					P42.4×2.6	30		
	P60.3×3.6	80					P48.3×2.6	72		
	P76.1×2.9	112					P48.3×3.2	72		
	P76.1×3.6	20					P60.3×2.9	36		
BV2	P101.6×4.5	4	446.89	P42.4×2.6	780	609.21	P33.7×2.6	212	218.53	1274.63
	P101.6×5.6	4								
	P108×5.6	8								
	P48.3×2.6	280		P48.3×2.6	48		P42.4×2.6	72		
	P48.3×3.2	16								
	P60.3×2.9	94					P48.3×2.6	60		
	P76.1×2.9	18		P48.3×3.2	20					
	P76.1×3.6	14					P48.3×3.2	48		
	P76.1×4	4								
	P76.1×5	22		P60.3×2.9	52		P60.3×2.9	28		
	P88.9×5	16								
BV3	P101.6×5.6	18	453.28	P42.4×2.6	748	632.37	P33.7×2.6	242	211.96	1297.61
	P108.0×5.6	4								
	P114.3×7.1	4		P48.3×2.6	64		P33.7×3.2	6		
	P133.0×6.3	4								
	P139.7×7.1	4		P48.3×3.2	24		P42.4×2.6	88		
	P48.3×2.6	354								
	P48.3×3.2	4		P48.3×4	16		P48.3×2.6	24		
	P60.3×2.9	62								
	P76.1×4	10		P60.3×3.6	40		P48.3×3.2	4		
	P76.1×5	8								
	P88.9×5	8		P60.3×5	8		P60.3×2.9	56		

Table 2 Structural properties of members BV4, BV5 and BV6

Model	Top Layer			Web Layer			Bottom Layer			Total Wight
	Section	Quantity	Layer Weight	Section	Quantity	Layer Weight	Section	Quantity	Layer Weight	
BV4	P48.3×2.6	314	381.61	P42.4×2.6	880	581.1	P33.7×2.6	274	203.79	1166.5
	P48.3×3.2	4					P42.4×2.6	46		
	P60.3×2.9	44					P48.3×2.6	60		
	P60.3×3.6	20					P48.3×3.2	14		
	P76.1×2.9	78		P48.3×2.6	20					
	P76.1×3.6	16					P60.3×2.9	26		
	P76.1×4	4								
BV5	P114.3×3.6	20	394.62	P42.4×2.6	784	607.18	P33.7×2.6	266	201.78	1203.58
	P114.3×4.5	4					P33.7×3.2	6		
	P114.3×5.6	4					P42.4×2.6	68		
	P48.3×2.6	354		P48.3×2.6	44		P48.3×2.6	44		
	P60.3×2.9	72								
	P60.3×3.6	8		P48.3×3.2	32		P48.3×3.2	16		
	P76.1×2.9	4								
	P76.1×3.6	4		P60.3×2.9	40		P60.3×2.9	20		
	P76.1×5	10								
BV6	P101.6×4.5	10	402.81	P42.4×2.6	732	629.97	P33.7×2.6	302	199.63	1232.41
	P101.6×5.6	8		P48.3×2.6	84		P33.7×3.2	8		
	P114.3×5.6	4		P48.3×3.2	24		P42.4×2.6	46		
	P114.3×7.1	4								
	P139.7×6.3	4		P48.3×4	16		P48.3×2.6	20		
	P48.3×2.6	376								
	P48.3×3.2	4		P60.3×3.6	40		P48.3×3.2	4		
	P60.3×2.9	56								
	P60.3×3.6	10		P60.3×5	4		P60.3×2.9	40		
	P76.1×2.9	4								

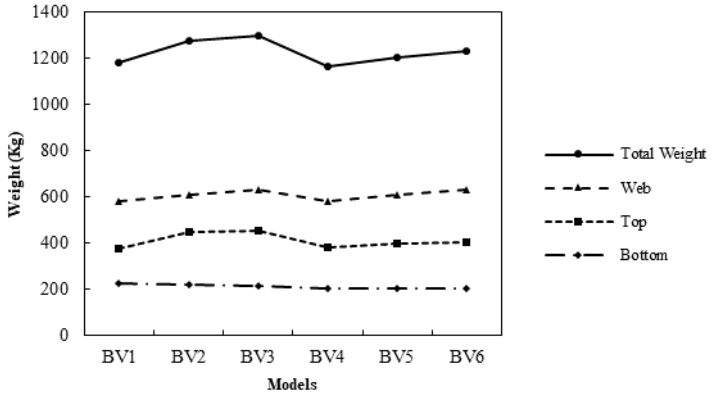


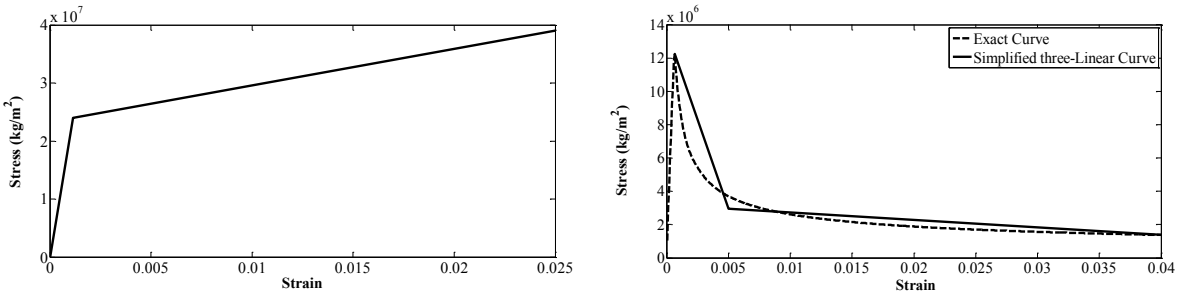
Figure.3 Comparison of weight for each layer and whole structure

Progressive collapse analysis

After designing the models with different support conditions, they were transferred to the OpenSees software for performing a progressive collapse analysis, (Mazzoni et al. 2006). Both material and geometrical nonlinearities are considered in models. Members were represented by truss elements using the corotational approach to model geometric nonlinearities (corotTruss element in OpenSees). For applying material nonlinearity effect, Hystertic Material was used,(Mazzoni et al. 2006).

Each truss element can sustain only axial, tensile or compression, load. Considering the possibility of buckling of member under compressive loads before reaching maximum compressive strength, the behaviour of the member will be asymmetric. To imply the nonlinear behaviour under compression load, the buckling behaviour is applied to material stress-strain curve in compression region. To reach this goal, a preliminary simple model including one beam-column element with initial imperfection is simulated. The model has pin support at one end and restricted in one direction and free in another direction at the other end. A displacement-based nonlinear static analysis under compressive load was performed for each cross section with the lengths compatible with assumed double layer barrel vault system. Figure (4-a) shows stress-strain curve of steel material for tensile branch considered as bilinear with a 3% of initial stiffness in post-yielding region. Figure (4-b) shows a typical stress-strain curve of steel material for compression branch resulted for P48.3×4.0 section with 2 (m) length. In this figure, the precise buckling behaviour of member and the simplified three-line behaviour which used in the modelling are illustrated.

The progressive collapse analysis of the models begins by applying gravity loads in a static load-controlled situation; at this step the critical member is determined based on the highest stress ratio under symmetric and asymmetric snow loading conditions within each top, web and bottom layer. Then, by keeping the loads constant, the analysis enters to the dynamic phase. At this stage, the critical member is removed from the structure in a very short time nonlinear dynamic analysis is started. According to the General Services Administration guidelines, the removal time of element should be less that 0.1 of the first natural period of the structure, (GSA 2016). The analysis of the structure is continued until the structure achieves a balance or is completely destroyed. At each time step, the component of displacement of nodes is recorded. The node which has the maximum spatial displacement considered as critical node. In the numerical models, Rayleigh method was used to create the damping matrix of structure. In this method represents the damping matrix with a linear combination of mass matrix and stiffness matrix.



(a) (b)

Figure.4 Stress-strain curve for members in (a) tension and (b) compression branch

Results

To investigate the occurrence of progressive collapse, displacement of critical node regarding to the time step is plotted. Furthermore, the stress and strain value of members is recorded in each time step to identify number of elements which may fail in tension or compression.

In this section, the results of the progressive collapse analysis for each model are presented. Figure (5) shows displacement of critical point at each time step under symmetric snow load condition. It also provides critical member's identification number which is removed from the structure. According to this figure progressive collapse occurs in model BV2 and BV3. In other models the progressive collapse under symmetric snow load is not probable and after failure of the critical member the structure reaches to balance after some vibration.

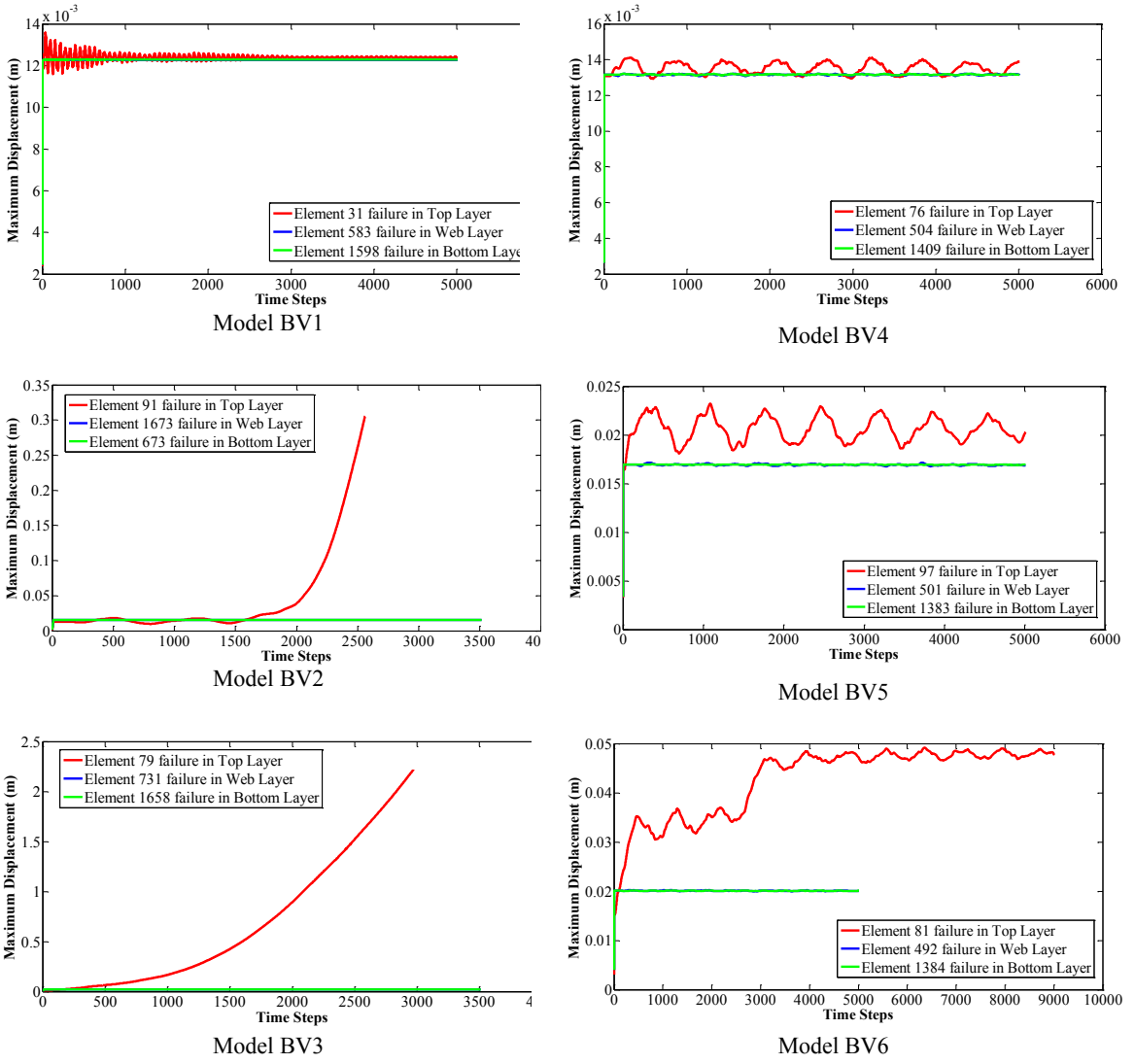


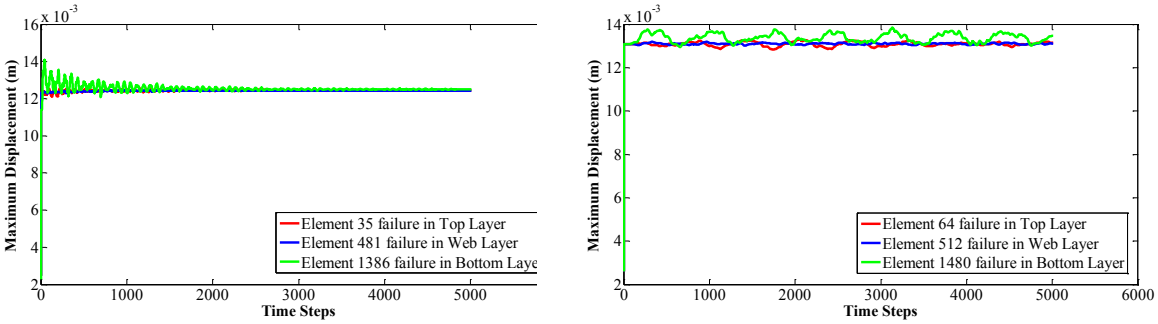
Figure .5 critical point displacement under symmetric snow load condition

Results show that under symmetric snow load condition, the structure is more sensitive to failures occurred in the top layer. In model BV6 reaches to initial stability after removal of critical member. However, after some vibration, some more members fail and the displacement of the critical load increases. However in this stage, the structure is resistant enough to reach final stability after some vibration. Table (3) summarizes the number of members failed during dynamic analysis in the whole structure. In this table DCR column, shows the Demand to Capacity ratio of the removed element, NMFC columns shows the Number of Members Failed in Compression, and NMFT shows the Number of Members Failed in Tension.

Table 3 Number of members failed under symmetric snow load condition

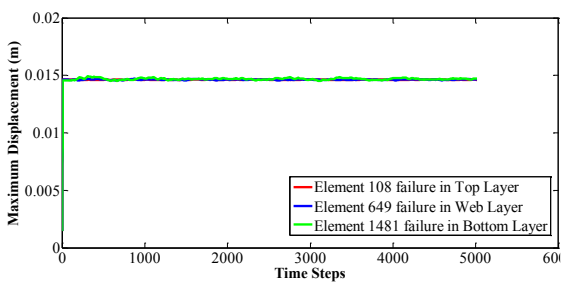
Model	Critical Member failure in Top Layer			Critical Member failure in Web Layer			Critical Member failure in Bottom Layer		
	Critical Member's DCR	NMFC	NMFT	Critical Member's DCR	NMFC	NMFT	Critical Member' s DCR	NMFC	NMFT
BV1	0.93	1	0	0.22	0	0	0.04	0	0
BV2	0.98	140	3	0.9	1	0	0.083	0	0
BV3	0.95	475	24	0.92	1	0	0.63	0	0
BV4	0.95	1	0	0.86	1	0	0.66	1	0
BV5	0.94	6	0	0.9	1	0	0.93	1	0
BV6	0.95	28	1	0.93	1	0	0.75	1	0

Figure (6) shows displacement of critical point at each time step under asymmetric snow load condition along with critical member's identification number which is removed from structure. According to this figure progressive collapse is not probable in all models under asymmetric snow load condition; because after failure of the critical member, the structure reaches to balance after some vibration. Also it is comprehensible that under asymmetric snow load condition, the structure is more sensitive to failures which occur in bottom layer.

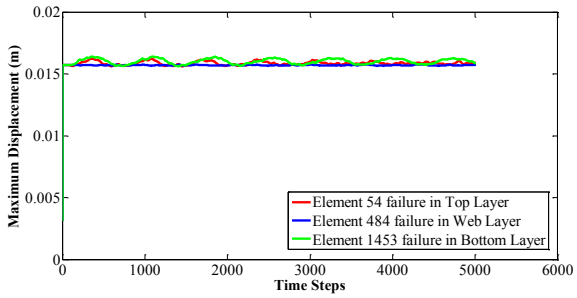


Model BV1 Model BV4

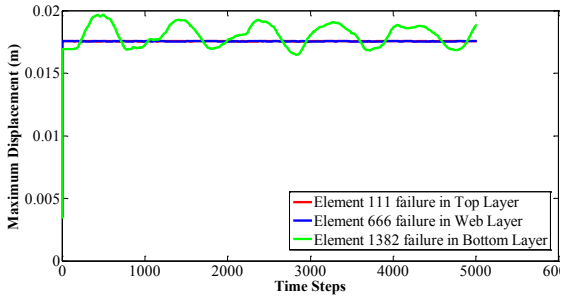
Figure .6 critical point displacement under asymmetric snow load condition



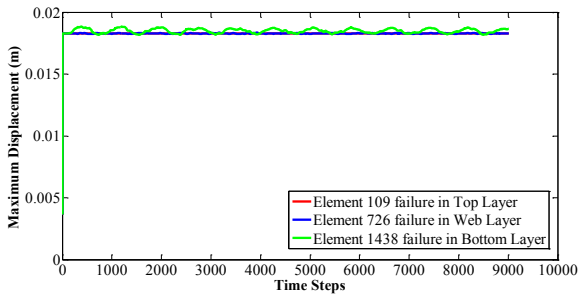
Model BV2



Model BV5



Model BV3



Model BV6

Figure.6 (continued) critical point displacement under asymmetric snow load condition

Table 4 Number of members failed under asymmetric snow load condition

Model	Critical Member failure in Top Layer			Critical Member failure in Web Layer			Critical Member failure in Bottom Layer		
	Critical Member's DCR	NMFC	NMFT	Critical Member's DCR	NMFC	NMFT	Critical Member's DCR	NMFC	NMFT
BV1	0.91	1	0	0.68	0	0	0.94	1	0
BV2	0.93	0	0	0.94	1	0	0.94	1	0
BV3	0.88	0	0	0.92	0	0	0.92	2	0
BV4	0.93	1	0	0.81	1	0	0.94	1	0
BV5	0.94	1	0	0.92	1	0	0.92	1	0
BV6	0.9	0	0	0.93	0	0	0.93	1	0

Conclusion

In the present study, the vulnerability of double-layer braced barrel vaults with different support conditions was evaluated under progressive collapse under symmetric and asymmetric snow load condition. Six numerical models with different support conditions were selected and designed. Then,

critical members based on their stress level have been identified. After removal of critical member from structure, the response of the structure is obtained using nonlinear dynamic analysis. The following results can be summarized as the conclusion of this research:

- Double layer barrel vault is more vulnerable to progressive collapse under the symmetric snow load with supports provided only in longitudinal direction.
- If the supports are considered only in longitudinal direction, the vulnerability of structures to progressive collapse increases by decreasing the density of supports.
- Double layer barrel vault is resistant to progressive collapse under asymmetric snow loading condition with any support configuration. This confirms that local accumulation of the loads cannot lead to a total failure of this structure due to their specific load redistribution capability.
- In symmetric loading condition, the structure is more sensitive to failures occurring in top layer; however, in asymmetric loading condition the structure is sensitive to failures occurring in bottom layer.
- In order to design double layer barrel vaults economically efficient and resistant to progressive collapse, it is suggested that supports should be considered in both longitudinal and transvers direction with minimum possible distance.

References

American society of civil engineers. 2014. *Asce 7-16*. *Science*. Vol. 69. https://doi.org/10.1126/science.69.1782.217-a.

ANST/AISC. 2016. "AISC 360-16: Specification for Structural Steel Buildings ANSI/AISC 360-16," 676. https://doi.org/111.

Arabi, Saeed. 2017. "Vulnerability Assessment of Sign-Support Structures during Transportation and in Service." *Graduate Theses and Dissertations*. Ames: Iowa State University, Digital Repository. https://doi.org/10.31274/etd-180810-6133.

Arabi, Saeed, Behrouz Shafei, and Brent M. Phares. 2018. "Fatigue Analysis of Sign-Support Structures during Transportation under Road-Induced Excitations." *Engineering Structures* 164 (June 2017): 305–15. https://doi.org/10.1016/j.engstruct.2018.02.031.

Arabi, Saeed, Behrouz Shafei, and Brent M. Phares. 2019. "Investigation of Fatigue in Steel Sign-Support Structures under Diurnal Temperature Changes." *Journal of Constructional Steel Research* 153 (September): 286–97. https://doi.org/10.1016/j.jcsr.2018.09.024.

Arabi, Saeed, Behrouz Shafei, and Brent M Phares. 2017. "Vulnerability Assessment of Sign Support Structures During Transportation." https://trid.trb.org/view/1439268.

Chilton, John (John C.). 2000. *Space Grid Structures*. Architectural Press. .

CSI. 2015. "CSI Analysis Reference Manual," no. December: 528.

GSA. 2016. "General Services Administration Alternate Path Analysis & Design Guidelines for Progressive Collapse Resistance," 1–203.

Kave, A, and B Eftekhar. 2012. "Optimal design of double layer barrel vaults using an improved hybrid big bang-big crunch method." *Asian Journal of Civil Engineering*. https://www.researchgate.net/publication/289265310_Optimal_design_of_double_layer_barrel_vaults_using_an_improved_hybrid_Big_Bang-Big_Crunch_method.

Kaveh, A., M. Farahani, and N. Shojaei. 2012. "Optimal Design of Barrel Vaults Using Charged Search System." *International Journal of Civil Engineering* 10 (4): 301–8. http://ijce.iust.ac.ir/browse.php?a_id=612&sid=1&slc_lang=fa.

Kaveh, A., and S. Mahjoubi. 2018. "Optimum design of double-layer barrel vaults by lion pride

optimization algorithm and a comparative study." *Structures* 13 (February): 213–29. https://doi.org/10.1016/J.ISTRUC.2018.01.002.

Kaveh, A., B. Mirzaeib, and A. Jafarvandb. 2013. "Optimal design of double layer barrel vaults using improved magnetic charged system search." https://www.semanticscholar.org/paper/optimal-design-of-double-layer-barrel-vaults-using-kaveh-mirzaeib/60a400dca4871da94572cf5a12a78fddcd1275d7.

Kaveh, A., and M. Moradveisi. 2016. "Optimal design of double-layer barrel vaults using cbo and ecbo algorithms." *Iranian Journal of Science and Technology, Transactions of Civil Engineering* 40 (3): 167–78. https://doi.org/10.1007/s40996-016-0021-4.

Kramer, C., and H. J. Gerhardt. 1988. *Advances in Wind Engineering. Part 1□: Proceedings of the 7th International Conference on Wind Engineering Held under the Auspices of the International Association for Wind Engineering, Aachen, F.R.G., July 6-10, 1987*. Elsevier.

Makowski, Z. S. 1990. *Analysis, Design and Construction of Braced Barrel Vaults*.

Martin, Rachel, and Norbert J. Delatte. 2001. "Another Look at Hartford Civic Center Coliseum Collapse." *Journal of Performance of Constructed Facilities* 15 (1): 31–36. https://doi.org/10.1061/(ASCE)0887-3828(2001)15:1(31).

Mazzoni, Silvia, Frank McKenna, Michael H Scott, and Gregory L Fenves. 2006. "OpenSees Command Language Manual." *Pacific Earthquake Engineering Research Center*, 451.

Nooshin, Hoshyar, and Peter Disney. 2002. "Formex Configuration Processing III." *International Journal of Space Structures* 17 (1): 1–50. https://doi.org/10.1260/026635102760123042.

Shin, Jiuk, Kihak Lee, Gee-Cheol Kim, Chan-Woo Jung, and Joo-Won Kang. 2012. "Analytical and Experimental Studies on Seismic Behavior of Double-Layer Barrel Vault Systems with Different Open Angles." *Thin-Walled Structures* 54 (May): 113–25. https://doi.org/10.1016/J.TWS.2012.02.008.

Subramanian, N. (Narayanan). 1999. *Principles of Space Structures*. Wheeler.

Tahamouli Roudsari, M., M. Gordini, H. Fazeli, and B. Kavehei. 2017. "Probability Analysis of Double Layer Barrel Vaults Considering the Effect of Initial Curvature and Length Imperfections Simultaneously." *International Journal of Steel Structures* 17 (3): 939–48. https://doi.org/10.1007/s13296-017-9006-9.

Yadollahi, Mehrzad Mohabbi. 2011. "The effect of initial geometric imperfection on the load bearing capacity of double layer barrel vault space structures."