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Applying Multi-objective Geometric Programming to the Fuzzy Inventory of Kerosene in Al-Dora Refinery

Ruqayah Yasser Abdul Amir [a]

[a] *Lecturer, Department of Banking and Financial Sciences, Imam Al-Kadhum College, Iraq.*
E-mail: rukaya.yasser@alkadhum-col.edu.iq

Abstract

A multi-objective inventory model was formulated by decreasing the unit cost on which order depends. The model was formulated with three constraints: storage space, number of orders, and production cost. We note in the research that the cost parameters in the objective function and constraints are imprecise in nature. Hence, the cost criteria are superimposed on the objective function and constraints in an ambiguous environment. Geometric programming was used to solve the production model without fuzzy deficit. It turned out that this model is better, as different possibilities were taken, and the total cost at these possibilities was shown. The best order volume was (1544117.647) liters, because it achieved the lowest total cost of storage, which amounted to (94,577,205.91) Iraqi dinars.

Keywords: Geometric Programming, Fuzzy Inventory, Storage Model without Deficit.

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Introduction

The inventory problem is considered one of the most important problems facing the management of any institution or company. One of the most important of these problems is how to take the appropriate decision represented in maximizing profits, reducing costs, controlling inventory, or increasing production. Therefore, the storage function is considered one of the most important jobs approved in companies and institutions. It plans and organizes the storage and preservation of materials and commodities and supplies the requesting authorities with their needs in a timely manner. If the amounts of orders for a commodity are fixed and that the price of one unit is also fixed and independent, in this case, the storage problem will be classic and routine. In practice, the amount of order and the price per unit are linked to each other. When the demand for a commodity is high, that commodity will be produced in large numbers. Consequently, this will result in a high production cost and the cost per unit price. The unit will decrease, indicating that the price per unit is inversely related to demand. Therefore, demand can be considered a decision variable. In my research, I formulated the problem of multi-item, multi-objective inventory along with three constraints, namely, storage space, number of orders, and production cost. Deficits are allowed except for accumulative, cost parameters and constraints are considered in the search under an uncertain environment. The research problem was addressed by using geometric programming (GP) method.

Symbols Used

The parameters for items (n=1,2,…n) are-:
D_i order per unit(^{t})Dn,...,D_2,D=(D_1)
Q_i economic quantity) ^{t})Qn,...,Q_2,Q=(Q_1)
(S=(S_1,$S_{2,...,}S_n$)^t) storage level S_i
The cost of holding inventory per unit c_{1i}
Deficit cost per unit c_{2i}
The cost of preparing the order c_{3i}
m_i storage space
Total cost $TC_i(D_i, Q_i, S_i)$
Allowance for the total cost TC_{0i}
P_{0i} Allowance amount associated with the total cost
m Storage goal
P_m The amount of allowance related to the storage space
T goals associated with the number of units required
P_t Allowance amount associated with required units
U goals associated with the cost of production
P_u The amount of allowance associated with the cost of production
n the number of items

The Research Problem

Through my observation of the nature of the work in Al-Dora Refinery, I found that the quantities of production, the amount of order for products, and the change in the cost of production are fuzzy. In addition, there is an inaccuracy of the constraints associated with the problem, which are the storage space, investment, and the number of orders. This led to fuzziness in the quantities produced. The change in order quantities is related to the changing need for these products, leading to a change in the amount of profitability. Therefore, I decided to use geometric programming to address this problem.

The Research Objective

The aim of the research is to address the problem of fuzzy multi-unit and multi-objective inventory in Al-Dora refinery with regard to production quantities and product ordering quantities, as well as knowing the amount of elasticity and its impact on the total cost, and finding the optimal solution by using geometric programming.

Theoretical Side

Geometric Programming

It is a mathematical method for solving non-linear programming problems. It uses min. function, which is originally subject to real constraints, which are positive polynomials. It is used to address some geometric design problems, as it reduces the cost to a minimum (1). Geometric programming has the ability to convert the primal objective function, which is a reduction of the function into positive polynomial functions, which are the result of multiplying all the

constraints of the cost function and the constraints in a special way called the dual function. In this case, the function is maximized. (Liu, et al., 2006). (Jung, & Klein, 2001)

Geometric Programming Problem with Constraints

Most of the optimal geometric programming problems are subject to multiple constraints. Geometric programming adopts the following formula:

$$g_k(x) = \sum_{i\in[k]} c_i \prod_{j=1}^{m} x_j^{a_{ij}} \quad \leq 1 \ldots..(1)$$

where

$$c_i, x_j > 0$$

Through equation No. (1) above, the multi-objective geometric programming problem (general formula) can be formulated as follows:

$x = (x_1, x_2, x_3, \ldots x_n)$

$$min.\, g_{k0}(x) = \sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}} \quad , \quad k = 1,2,3, \ldots, p$$

Subject to

$$g_i(x) = \sum_{t=1}^{T_i} c_{it} \prod_{j=1}^{n} x_j^{d_{itj}} \leq 1 \quad i = 1,2,3, \ldots m$$

$$x_j > 0 \quad , \quad j = 1,2,3, \ldots n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

c_{k0t} Positive real numbers for each t, k p number of functions

d_{itj} Real numbers for i, j, t, km is the number of constraints

T_{k0} The number of units in k of the objective functions

n number of decision variables

T_i The number of units for i of the constraints(

(M.O. Abou-el-ata,2003),(S.Islam,2006)

Formulation of Inventory Model with Deficit

There are many hypotheses when applying this model, which are as follows:

Deficits are permitted with constraints on the average total cost of the deficit. (Hartley, (1978), Soni et al. (2015), Luo, (2003)

The production rate is immediate and inversely related to the demand rate, as it takes the following form:

$$c_{4i} = \alpha_i D_i^{-\beta_i} \alpha_i > 0 \beta_i > 1$$

The amount of inventory for a number of *ith* items (i=1,2,3,…n). The storage level, Ri, is at time t=0. During the time period $T_i = (t_{1i} + t_{2i}), 0$ the stock level decreases gradually to meet customer's demands, as the stock level reaches zero in the period T_i, t_{1i} and then a deficit is allowed to occur in the period from T_i, t_{1i}. The same cycle is repeated by this storage process shown in Figure (1)

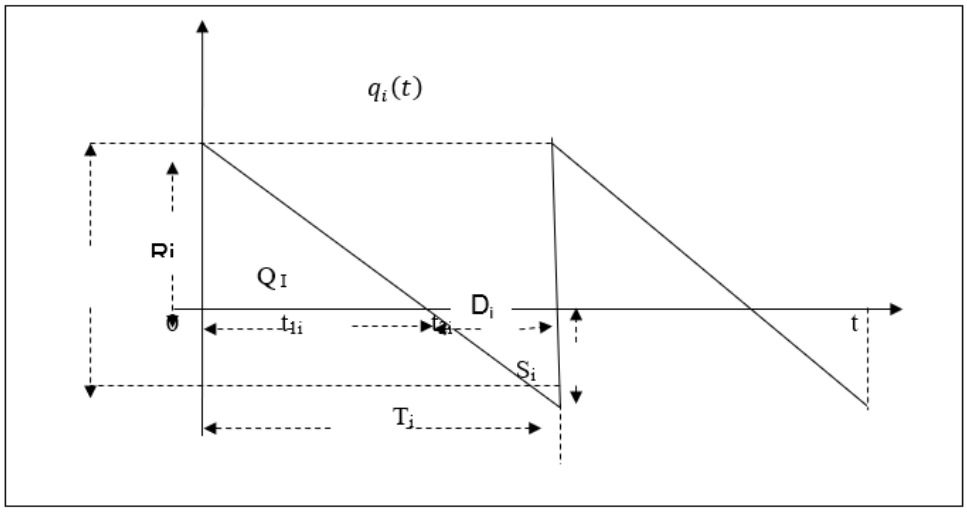


Fig. 1: Shows the Storage Level

Quick inventory $q_i(t)$ during t (time period) T_i,0 can be shown in the following differential equations:

$$\frac{d_{qi}(t)}{d_t} = \begin{cases} -D_i \; in \quad 0 \le t \le t_{1i} \\ -D_i \; in \quad t_{1i} \le t \le t_{1i} + t_{2i} \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

With conditions and determinants, namely:

$$q_i(t) = \begin{cases} Q_i - S_i \text{ at } t=0 \\ 0 \quad \text{at } t=t_{1i} \\ -S_i \quad \text{at } T_i=t_{1i} + t_{2i} \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

In each of the preceding time periods a fixed amount of deficit is allowed. There is a penalty cost (shortage cost) $c_{2i}$for the order component that is not met per unit time.

$$q_i(t) = \begin{cases} R_i - D_{it} 0 \le t \le t_{1i} \\ D_i(t_{1i} - t) t_{1i} \le t \le t_{1i} + t_{2i} \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

So, from figure (1), we conclude the following:

$D_i t_{1i} = R_i D_i t_{2i} = S_i D_i T_i = Q_i$

We take the integration of equations (2) and (3) to obtain the cost of holding and the cost of deficit in order to write the total cost of storage as follows:

The cost of holding stock $= c_{1i} \int_0^{t_{1i}} q_i(t) = C_{1i} \frac{(Q_i - S_i)^2}{2Q_i} T_i$……..(4)

Cost of defici $= c_{2i} \int_{t_{1i}}^{T_i} (-q_i(t)) dt = c_{2i} \frac{S_i^2}{2Q_i} T_i$ ……(5)

Cost of production $= c_{4i} Q_i = \alpha_i D_i^{-\beta_i} Q_i$ ……(6)

The cost of preparing the order $= c_{3i} \frac{D_i}{Q_i}$ ……..(7)

Total cost of storage (C.T) = cost of production + cost of preparing his order + cost of maintaining storage + cost of deficit

$$T.C = c_{4i} Q_i + c_{3i} \frac{D_i}{Q_i} + \frac{C_{1i}(Q_i - S_i)^2}{2Q_i} T_i + \frac{c_{2i} S_i^2}{2Q_i} T_i \ldots\ldots\ldots\ldots\ldots(8)$$

(8) is a cost minimization function

There are some constraints on the available resources that greatly affect the total cost of inventory and total costs are constraints on storage space. This constraint aims at controlling the space of the inventory and limit the facility need to build or rent special buildings for the purpose of maintaining the storage and related expenses, taking into account the proportionality between the stored quantity and the space available for storage and all its conditions required by the storage.

$$W_i Q_i \le W$$

Economic quantity. Q_i represents the storage space for each commodity from the storage, W_i, the maximum storage area W. constraints on the number of applications during the year.

The importance of this constraint is due to controlling the number of orders commensurate with the capacity of the establishment. The increase in the number of orders directly leads to an increase in the total cost.

This limitation can be formulated as follows:

$$\frac{D_i}{Q_i} \le b$$

Economic quantity. W_i represents the annual demand rate, Q_i, the maximum number of orders during a specified period of time.

3- The amount of investment in the total cost of production cannot be unlimited, as it may have a higher limit on the maximum investment in purchasing materials and storing them to meet the critical cases for which this storage was placed. Therefore, the mathematical relationship will be in the following form:

$$\alpha_i D_i^{-\beta_i} Q_i \le u$$

Q_i stands for the economic quantity. u stands for the largest amount of investment.

The problem lies in finding the optimization for each of the level of demand D_i the economic quantity Q_i and the amount of deficit S_i by reducing the total costs of the objective function shown in equation (8) taking into account the total area, the number of orders and the amount of investment from the total cost of production and within specific conditions. The form can be written as follows:

$$T.C(D_i, Q_i, S_i) = \alpha_i D_i^{-\beta_i} + c_{3i} \frac{D_i}{Q_i} + \frac{C_{1i}(Q_i - S_i)^2}{2Q_i} + \frac{c_{2i} S_i^2}{2Q_i} \quad \ldots\ldots\ldots\ldots\ldots(9)$$

S.T

$$W_i Q_i \le W$$
$$\frac{D_i}{Q_i} \le b$$
$$\alpha_i D_i^{-\beta_i} Q_i \le u$$
$$L_{D_i} \le D_i \le U_{D_i},\ L_{Q_i} \le Q_i \le U_{Q_i},\ L_{S_i} \le S_i \le U_{S_i}$$

The Fuzzy Set

Fuzzy or fuzzy logic can be defined as a way to determine the degree of membership. Here things cannot be classified into false and true, but there is another value between these two values. The fuzzy set was developed by the researcher (Zadeh), where he explained that the fuzzy set is a set of pairs of items arranged with the degree of their membership. Each item has a degree of membership whose values range from zero to one. The degree of membership of the item (X) to group A can be in the form of ordered pairs. (Al-Tai, (2007), Muhammad, (2007), Jayalakshmi & Pandian, (2012)

$\tilde{A} = \{(M_{\tilde{A}}(X), (X)); \forall X \in X\} \quad i = 1,2, \dots, n$

$M_{\tilde{A}}(X) \in [0,1]$

Membership Functions

The membership function is of great importance in defining Fuzzy Set Theory. They are the functions through which the Membership Degree of the item is calculated to the fuzzy set. The degree of membership represents one of the members of the ordered pair that represents the fuzzy set. Each ordered pair in the fuzzy set has a membership score of [0,1]. The membership function of the fuzzy set can be explained as follows:

$$M_{\tilde{A}}(X) \in [0,1] \quad \text{.....(10)}$$

That is, when the degree of membership to the item is equal to 1, it indicates that the item affiliates completely to the fuzzy set. When its degree of membership is equal to 0, this indicates that the item is not a member at all to the fuzzy set. As for the rest of the degrees that vary between zero and one, their interpretation would be "When the degree of membership approaches (0.5), it would indicate that the item belongs in relation to the fuzzy set and does not belong in the same relation. This item is called the equilibrium point. However, when the degree of membership is close to 0.9, it would indicate that the item is of membership to the fuzzy set in a percentage and does not belong to it in a percentage, and it is closer to affiliation than not. (A. Srinivasan, 2013), (D. Dutta, Pravan Kumar)

The Triangular Fuzzy

The membership functions can be divided into linear affine functions and nonlinear affine functions. The focus of our research will be about the linear affine functions, namely

Trigonometric functions: They are functions with three parameters $\tilde{A} = (a_1, a_2, a_3)$. Dutta & Kumar (2012) Kotb et al. (2011)

where a_1 and a_3 stand for the maximum and minimum limit of the membership function $\tilde{A}$ with a_2 It represents the mode or mean, which represents the fuzzy set of the number line R of the membership function.

$M_{\tilde{A}}(X): R \to [0,1]$

It can be expressed in the following way:

$$M_{\tilde{A}}(X) = \begin{cases} = \dfrac{X - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ = \dfrac{X - a_3}{a_2 - a_3} & a_2 \le x \le a_3 \\ 0 & x > a_3, x \le a_1 \end{cases} \quad \dots\dots(11)$$

Where

The range is on the left: a_1

It represents the mean or mode : a_2

The range is on the right: a_3

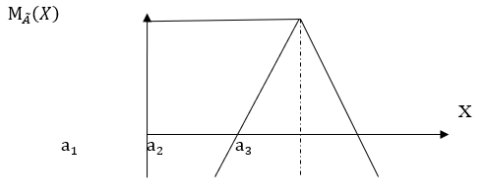


Fig. 2: Represents the Trigonometric Affine Function

Application

Some institutions, whose work is based on production, storage and even transportation, seek to benefit from a special discount or reduce the cost of storage or transportation by increasing the storage space if the situation requires. In this case, the inventory capacity (the space allocated for storage) becomes fuzzy, as well as the number of orders and the total production costs (investment amounts) are all inaccurate. In addition, the deficit costs, storage and production costs are also fuzzy and unclear, and thus the costs are, c_{1i} c_{3i}, c_{2i} in the objective function. They are fuzzy numbers, the total area, the total number of orders, and the cost of production are all fuzzy totals. With the aim of determining the level of products that Al-Dora refinery must maintain in order to ensure the continuation of its operations without interruption. Decision-making in this regard depends on the model that balances the cost of maintaining the storage against the loss resulting from the existence of a deficit (because of its impact on the supply and provision of liquid gas to citizens permanently). The model that was formulated contains only three fuzzy constraints, which are the storage space, the number of orders, and investment. The fuzziness is of the triangular type based on the available data from Al-Dora refinery.

1- Fuzzy Inventory with Imprecise Cost and Resources

Returning to the traditional model mentioned in Equation No. (9), and depending on the fuzzy set theory, the costs will become, $\widetilde{c_{1i}}$ $\widetilde{c_{2i}}$, $\widetilde{c_{3i}}$ Here, the definition is made by the membership functions, which may be linear or non-linear. In our research, linear functions were adopted: the functions of membership to the objective function, and the three constraints are:

$$T.C(D_i, Q_i, S_i; \widetilde{c_i}) = \alpha_i D_i^{-\beta_i} + \widetilde{c_{3i}} \frac{D_i}{Q_i} + \frac{\tilde{C}_{1i}(Q_i - S_i)^2}{2Q_i} + \frac{\tilde{c}_{2i} S_i^2}{2Q_i} \quad \ldots\ldots\ldots\ldots\ldots(12)$$

S.T

$$W_i Q_i \le W = \begin{cases} 0 & , \sum_{i=1}^{n} W_i Q_i > W + P_w \\ 1 - \sum_{i=1}^{n} \frac{W_i Q_i - W}{P_w} & , w \le \sum_{i=1}^{n} W_i Q_i \le W + P_w \end{cases}$$

$$, \sum_{i=1}^{n} W_i Q_i < W$$

$$\frac{D_i}{Q_i} \le b = \begin{cases} 0, \sum_{i=1}^{n} \frac{D_i}{Q_i} > t + P_t \\ 1 - \sum_{i=1}^{n} \frac{\frac{D_i}{Q_i} - t}{P_t} & , t \le \sum_{i=1}^{n} \frac{D_i}{Q_i} \le t + P_t \\ 1 & , \sum_{i=1}^{n} \frac{D_i}{Q_i} < t \end{cases}$$

$$\alpha_i D_i^{-\beta_i} Q_i \le u = \begin{cases} 0 & , \sum_{i=1}^{n} \alpha_i D_i^{-B_i} Q_i > u + P_u \\ 1 - \sum_{i=1}^{n} \frac{\alpha_i D_i^{-B_i} Q_i - u}{P_u} & , w \le \sum_{i=1}^{n} \alpha_i D_i^{-B_i} Q_i \le u + P_u \\ & , \sum_{i=1}^{n} \alpha_i D_i^{-B_i} Q_i < u \end{cases}$$

Where

Permitted violations of the total area: P_w

P_t : Violations allowed for the total number of orders

P_u : Permissible violations of total production costs

2- Apply the GP technique to solve fuzzy inventory model

In this research, data were collected from the General Company for the Distribution of Oil Derivatives - Al-Dora Production Refinery. Only one product was taken, which is (Kerosene) for the year 2021.

Table 1: Below Shows the Details of the Data, Noting that the Amounts are in the Tens of Thousands

Item	Total storage (W) space	One-unit storage space (ω_0)	Storage cost (H)	Production rate (a)	The cost of production per unit (P)
Kerosene	20,000	100	30	7	$9D^{-3}S^{-1}$

The optimal production quantity of kerosene is determined by the following formula:

$$q^* = \frac{W}{\omega_0} = \frac{20{,}000}{100} = 200$$

Determine the weights used in the geometric programming model:

$$\omega_1 = \frac{1}{4 - x} = \frac{1}{4 - 1.75} = \frac{1}{2.25}$$

$$\omega_2 = \frac{2-x}{4-x} = \frac{2-1.75}{4-1.75} = \frac{0.25}{2.25}$$
$$\omega_3 = \frac{1}{4-x} = \frac{1}{4-1.75} = \frac{1}{2.25}$$
$$\omega_{01} = \frac{2x-3}{4-x} = \frac{2(1.75)-3}{4-1.75} = \frac{0.5}{2.25}$$
Thus, we get the optimal solution
$$d^*(\omega) = (4-x)^{\omega_1} * \left[\frac{aH(4-x)}{6(2-x)}\right]^{\omega_2} * [\theta(4-x)]^{\omega_3} * \left[\frac{\omega_0(4-x)}{W(2x-3)}\right]^{\omega_{01}} * \left[\frac{2x-3}{4-x}\right]^{\omega_{01}}$$
After substituting the weights into $d^*(\omega)$ we get:
$$d^*(\omega) = 3.187$$
Formulating an inventory model (production without a deficit) for one commodity using geometric programming (G.P).
$$Max\ d(\omega) = \left(\frac{1}{\omega_1}\right)^{\omega_1} * \left[\frac{aH}{6\omega_2}\right]^{\omega_2} * \left[\frac{\theta}{\omega_3}\right]^{\omega_3} * \left[\frac{\omega_0}{W\omega_{01}}\right]^{\omega_{01}} * [\omega_{01}]^{\omega_{01}}$$
$$S.to$$
$$\omega_1 + \omega_2 + \omega_3 = 1$$
$$\omega_1 - \omega_3 = 0$$
$$\omega_1 - \omega_2 + (1-x)\omega_3 = 0$$
$$-\omega_1 + 2\omega_2 + \omega_{01} = 0$$
$$\omega_1, \omega_2, \omega_3, \omega_{01} \geq 0$$
Thus, we get the optimal solution for the cost of issuing the order (S), the optimal demand rate (D), and the total cost through:
$$S^* = \frac{6\omega_1\omega_2[d^*(\omega)]^2}{aH}$$
$$S^* = \frac{6\left(\frac{1}{2.25}\right)\left(\frac{0.25}{2.25}\right)[522.628\,]^2}{60000 * 30} = 0.014$$
$$D^* = \frac{aHq^2}{6\omega_2 d^*(\omega)}$$
$$D^* = \frac{60000 * 30 * 200^2}{6 * \frac{0.25}{2.25} * 522.628} = 495{,}755.56$$
$$TAC = \frac{SD}{q} + \frac{aHq^2}{6D} + \theta D^{1-x} S^{-1}$$
$$TAC = \frac{0.045 * 206{,}647{,}940.8}{200} + \frac{60000 * 30 * 200^2}{6 * 206{,}647{,}940.8} + 90000(206{,}647{,}940.8)^{1-1.75}(0.045)^{-1}$$
$$TAC = 37.561\ dinar$$

Processing Fuzzy Data

In this research, the trigonometric function was used to treat the fuzzy data related to the production rate, the cost of maintaining storage, as well as the storage space, as follows:
$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}{}^{L}(x), & a_1 < x < a_2 \\ \mu_{\tilde{A}}{}^{U}(x), & a_2 < x < a_3 \\ 0 & ,otherwise \end{cases}$$

Table 2: Below Shows the Fuzzy Data and how to Process it by Extracting the Lower and Upper Constraint for Each Parameter

total storage space (W)	(θ) production parameter	The cost of keeping the stock (H)	Production rate (a)
10000,25000,40000	10,15,17	35,25,40	5,3,13

$$a_L = \frac{a_1 + a_2}{2} = \frac{5+3}{2} = 4$$
$$a_U = \frac{a_2 + a_3}{2} = \frac{3+13}{2} = 8$$
$$H_L = \frac{H_1 + H_2}{2} = \frac{35+25}{2} = 30$$

$$H_U = \frac{H_2 + H_3}{2} = \frac{25 + 40}{2} = 32.5$$
$$\theta_L = \frac{\theta_1 + \theta_2}{2} = \frac{10 + 15}{2} = 12.5$$
$$\theta_U = \frac{\theta_2 + \theta_3}{2} = \frac{15 + 17}{2} = 16$$
$$W_L = \frac{W_1 + W_2}{2} = \frac{10000 + 25000}{2} = 17500$$
$$W_U = \frac{W_2 + W_3}{2} = \frac{25000 + 40000}{2} = 32500$$

4-4 Building a production model without fuzzy deficit using geometric programming:

$$Max\ d(\omega) = \left(\frac{1}{\omega_1}\right)^{\omega_1} * \left[\frac{({a_L}^{1-s}{a_U}^{s})({H_L}^{1-s}{H_U}^{s})}{6\omega_2}\right]^{\omega_2} * \left[\frac{{\theta_L}^{1-s}{\theta_U}^{s}}{\omega_3}\right]^{\omega_3} * \left[\frac{\omega_0}{({W_L}^{1-s}{W_U}^{s})\omega_{01}}\right]^{\omega_{01}} * [\omega_{01}]^{\omega_{01}}$$

$S.to$

$\omega_1 + \omega_2 + \omega_3 = 1$

$\omega_1 - \omega_3 = 0$

$\omega_1 - \omega_2 + (1 - x)\omega_3 = 0$

$-\omega_1 + 2\omega_2 + \omega_{01} = 0$

$\omega_1, \omega_2, \omega_3, \omega_{01} \geq 0$

Thus, we obtain the optimal solution for the model parameter

$$d^*(\omega) = (4 - x)^{\omega_1} * \left[\frac{({a_L}^{1-s}{a_U}^{s})({H_L}^{1-s}{H_U}^{s})(4-x)}{6(2-x)}\right]^{\omega_2} * \left[({\theta_L}^{1-s}{\theta_U}^{s})(4 - x)\right]^{\omega_3} * \left[\frac{\omega_0(4-x)}{({W_L}^{1-s}{W_U}^{s})(2x-3)}\right]^{\omega_{01}} * \left[\frac{2x-3}{4-x}\right]^{\omega_{01}}$$

Then we find the value of each of the cost of issuing the optimal order, the optimal production quantity, the optimal demand rate for kerosene, and the total storage cost, depending on the following formulas:

$$q^* = \frac{({W_L}^{1-s}{W_U}^{s})}{\omega_0}$$
$$S^* = \frac{6\omega_1\omega_2[d^*(\omega)]^2}{({a_L}^{1-s}{a_U}^{s})({H_L}^{1-s}{H_U}^{s})}$$
$$D^* = \frac{({a_L}^{1-s}{a_U}^{s})({H_L}^{1-s}{H_U}^{s})q^2}{6\omega_2 d^*(\omega)}$$
$$TAC = \frac{SD}{q} + \frac{({a_L}^{1-s}{a_U}^{s})({H_L}^{1-s}{H_U}^{s})q^2}{6D} + ({\theta_L}^{1-s}{\theta_U}^{s})D^{1-x}S^{-1}$$

After compensating the values a_L ،a_U ، H_L ،H_U ، θ_L ، θ_U ، W_L ، W_U and probability value (s) in $d^*(\omega)$ we get:

s	D^*	S^*	q^*	TAC
0.0	1544117.647	0.032	175	94577205.91
0.1	1877074.328	0.029	186.176	140583980.5
0.2	2281904.192	0.028	198.065	208977747.9
0.3	2774137.752	0.026	210.713	310655418.9
0.4	3372664.568	0.024	224.169	461819571.2
0.5	4100460.402	0.023	238.485	686562500
0.6	4984123.417	0.021	253.714	1020433051
0.7	6060062.812	0.020	269.917	1517124169
0.8	7366533.726	0.019	287.154	2255046262
0.9	8954972.678	0.017	305.491	3352006191
1.00	10886297.57	0.016	325	4982749116

Conclusions

Through using the geometric programming in solving the production model without deficit, we notice that the total cost of storing the optimal quantity was (37.561) in tens of thousands of dinars. When compared with the use of geometric programming to solve the production model without fuzzy deficit, this model turned out to be better. Various probabilities were taken and the total cost of these probabilities was shown. The best order volume was (1544117.647) liters, because it achieved the lowest total cost of storage, which amounted to (94,577,205.91) Iraqi dinars.

Recommendations

1 Geometric programming is one of the best ways to solve unstable and volatile problems because it facilitates the efficient development of available resources. It can bridge the shortcomings and find the best solution close to the nature of the problem to remedy the deficit and reach the best possible solutions.
2 A production inventory model without a fuzzy deficit is recommended to be formulated in order to reduce the total cost and thus increase production.
3 It is necessary to conduct multiple studies on the production storage model without fuzzy deficit and apply it to all production companies and institutions in Iraq.
4 The production of products that are expensive to produce and exceed demand should be reduced because they cause large losses.
5 The products that make a profit in production should be relied upon because they have a positive return on revenue, in contrast to products whose production has a high cost that leads to losses and reduces revenues.

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